Preface

This manual of notes and worksheets was developed by Teri E. Christiansen at the University of Missouri-Columbia. The goal was to provide students in Math 1100 (College Algebra) a resource to be used for both taking notes and working problems for additional practice. The structure of the notes and assignments parallels the contents of the textbook *College Algebra*, Version 3, by Carl Stitz and Jeff Zeager (http://www.stitz-zeager.com/), with minor exceptions. Definitions and exercises were often inspired by the material in the textbook.

In most cases, it is far cheaper to purchase a printed version of this workbook from the MU bookstore than it is to print the materials at home. No royalties will be received by any of the authors or their employers from the sale of either the text or workbook.

The version of the textbook that has been modified specifically for Math 1100 at MU is available at:

http://www.math.missouri.edu/courses/math1100/CABook.pdf

0.1: Basic Set Theory and Interval Notation

Sets

A _________ is a well-defined collection of objects which are called the _________________ of the set. ("Well-defined" means that there is a rule that can determine whether something belongs to the set or not.)

We can describe sets in multiple ways:

Verbal Method	Roster Method	Set-Builder Method
Uses a sentence to define the set.	*Begin with* $\{$, *list each element only once, and end with* $\}$.	*A combination of the other two methods using a variable such as* x.
Let C be the set of all letters in the word 'MIZZOU'.		
Let D be the set of all letters in the word 'MISSOURI'.		

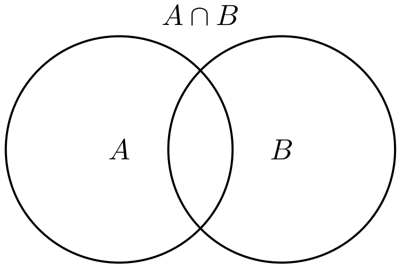
Notation

- $\in$ = "is an element of"
- $\notin$ = "is not an element of"
- $\subseteq$ = "is a subset of"
- $\not\subseteq$ = "is not a subset of"

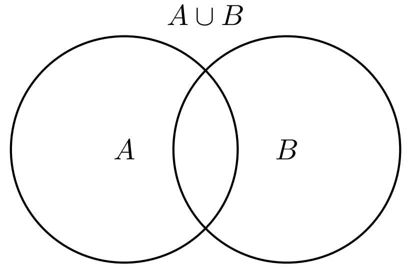
Intersection and Union

- The _________________of two sets A and B is the set: ______________________________________.
- The _________________of two sets A and B is the set: ______________________________________.

$A \cap B$



$A \cup B$



(EX) Consider the sets C and D listed above. Describe the sets below, using the roster method:

(a) $C \cap D$

(b) $C \cup D$

The **Empty Set**: $\emptyset = \{\}$. This is the set with no elements.

Given the sets

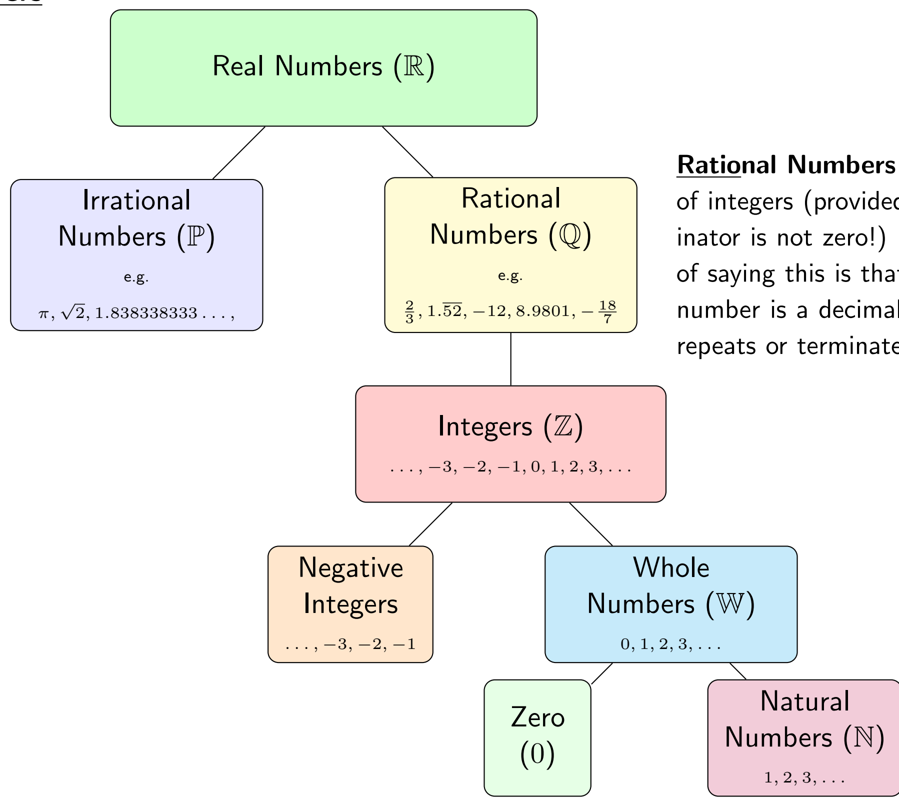
$$A = \{0, 2, 6, 7, 9\} \quad \text{and} \quad B = \{0, 7, 9, 10\}$$

find the following sets and write the answer using set notation. Separate multiple elements by commas. Write **empty** if the set contains no elements.

(a) $A \cap B =$

(b) $A \cup B =$

Sets of Real Numbers



An **Irrational Number** is a decimal which neither repeats nor terminates.

Rational Numbers are the ratios of integers (provided the denominator is not zero!) Another way of saying this is that a rational number is a decimal which either repeats or terminates.

Natural Numbers are sometimes called the counting numbers.

In addition, there is one other set which you should be familiar with:

- The **Complex Numbers**: $\mathbb{C} = \{a + bi \,|\, a, b \in \mathbb{R} \text{ and } i = \sqrt{-1}\}$ Despite their importance, the complex numbers play only a minor role in the text, and we will not cover them explicitly in this course.

(EX) Consider the set: $A = \left\{ -8.3\overline{3},\ \frac{17}{9},\ -\sqrt{5},\ -\frac{10}{5},\ \frac{0}{4},\ \sqrt{16},\ -2.84,\ \sqrt{\frac{4}{9}},\ \frac{\pi}{2} \right\}$. Which elements of A are:

(a) Integers?

(b) Rational numbers?

(c) Integers, but not whole numbers?

WeBWorK

Determine the validity of each statement by selecting True or False.

select ▾ **1.** The number $\sqrt{\frac{25}{100}}$ is rational, but not an integer

select ▾ **2.** The number $\sqrt{19}$ is a real number, but not an irrational number

select ▾ **3.** The number $0.80220200200020000 2\ldots$ is rational

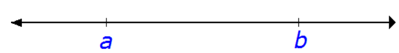
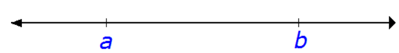
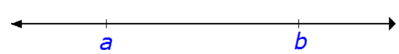
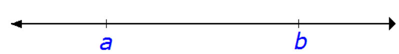
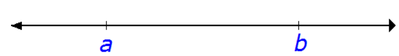
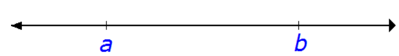
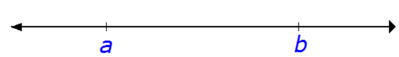
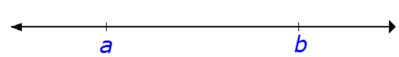
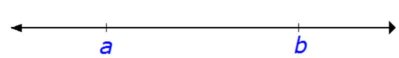
select ▾ **4.** The number $\sqrt{8}$ is a real number, but not a rational number

select ▾ **5.** The number $\sqrt{37^2}$ is a real number, but not a rational number

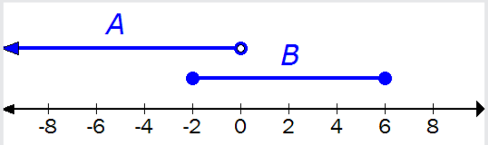
Interval Notation

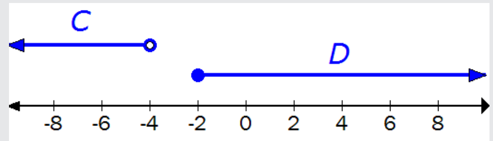
Segments of the real number line are called intervals. When we want to describe a set of numbers on the real number line, we use ____________________________. Here are a few guiding principles for using this notation:

- Each interval is written using the order 'left endpoint, right endpoint'.
- If an endpoint is **included** in the interval, we use a square bracket: [or] (These are called 'closed' intervals.)
- If an endpoint is **excluded** in the interval, we use parentheses: (or) (These are called 'open' intervals.)
- If an interval extends indefinitely to the left or right, we use $-\infty$ or ∞ as the endpoint, and use parentheses.

Set-Builder Notation	Interval Notation	Graph
$\{x \mid a < x < b\}$		
$\{x \mid a \le x < b\}$		
$\{x \mid a < x \le b\}$		
$\{x \mid a \le x \le b\}$		
$\{x \mid x < b\}$		
$\{x \mid x \le b\}$		
$\{x \mid x > a\}$		
$\{x \mid x \ge a\}$		
$\mathbb{R}$		

(EX) Given the sets shown on each graph below, find interval notation for each of the sets listed.





(a) A

(b) B

(c) $A \cup B$

(d) $A \cap B$

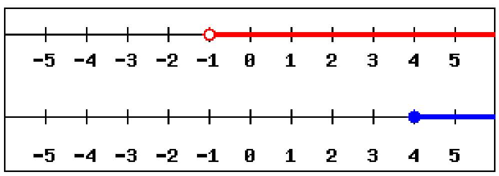
(e) C

(f) D

(g) $C \cup D$

(h) $C \cap D$

WeBWorK

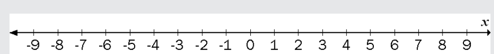
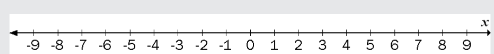
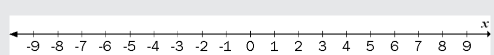
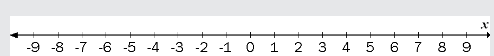


(a) Write the interval representing the **union** of the sets shown in the graphs.
Answer:

(b) Write the interval representing the **intersection** of the sets shown in the graphs.
Answer:

Help: Type **no solutions** if no such interval exists. Click here for help entering intervals as answers.

(EX) Complete the table below.

Set-Builder Notation	Interval Notation	Graph
$\{x \mid x \leq -5$ or $x > 2\}$		
$\{x \mid x \neq 2, 7\}$		
$\{x \mid x > 0$ and $x \neq 4\}$		
$\{x \mid x < -2$ or $x = 1\}$		

WeBWorK

Write in interval notation:

(a) $\{x \mid x \leq -74 \text{ or } x \geq -4\}$:

(b) $\{x \mid -74 \leq x < -4\}$:

Help: Click here for help entering intervals as answers.

0.2: Real Number Arithmetic

Properties of Real Numbers:

	Property of Addition	Property of Multiplication
Commutativity	For all real numbers a and b, $a + b =$ ________	For all real numbers a and b, $ab =$ ________
Associativity	For all real numbers a, b, and c, $a + (b + c) =$ ________________	For all real numbers a, b, and c, $a(bc) =$ ________________

Distributive Property: For all real numbers a, b, and c:

$a(b + c) =$ ________________ and $(a + b)c =$ ________________

Zero Product Property: For all real numbers a and b:

$ab = 0$ if and only if ________ or ________ (or both)

Working With Fractions:

- For a fraction $\frac{A}{B}$, A is called the *numerator* and B is called the *denominator*.
- Any integer can be written as a fraction by writing it with a denominator of 1: $7 = \frac{7}{1}$, $-2 = -\frac{2}{1}$.
- **Multiplication and Division:** (Common denominators are NOT needed!)
  - ▷ $\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D}$
  - ▷ $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{A \cdot D}{B \cdot C}$
- **Addition and Subtraction:** (Common denominators ARE needed!)
  - ▷ $\frac{A}{B} + \frac{C}{D} = \frac{A \cdot D}{B \cdot D} + \frac{C \cdot B}{D \cdot B} = \frac{AD + CB}{BD}$
  - ▷ $\frac{A}{B} - \frac{C}{D} = \frac{A \cdot D}{B \cdot D} - \frac{C \cdot B}{D \cdot B} = \frac{AD - CB}{BD}$
- **Reducing Fractions:** If a common factor can be factored out of the entire numerator and denominator, it may be canceled to give a fraction in *lowest terms*. (Note that you cannot cancel one part of a numerator/denominator when reducing a fraction - you must reduce a common factor from every term.)
- **Fractions Involving Zero:**
  - ▷ $\frac{0}{A} = 0$
  - ▷ $\frac{A}{0}$ is undefined
  - ▷ $\frac{0}{0}$ is called 'indeterminate'
- **Negatives and Fractions:**
  - ▷ $-\frac{A}{B} = \frac{-A}{B} = \frac{A}{-B}$
  - ▷ $\frac{-A}{-B} = \frac{A}{B}$

(EX) Perform the indicated operations and simplify.

(a) $\frac{2}{3} \cdot \frac{7}{5}$

(b) $21 \div -\frac{14}{9}$

(c) $\frac{2}{3} + \frac{7}{5}$

(d) $\frac{4}{9} - 4$

(e) $\left(\frac{11}{6} \cdot 2\right) + \frac{3}{4}$

WeBWorK

Perform the following operations if possible or write **undefined**. Give your answer as a fraction if necessary and simplify.

(a) $-\frac{3}{5} \div \left(-\frac{1}{7}\right) =$ ______

(b) $8 \div \left(-\frac{1}{5}\right) =$ ______

<u>**Properties of Integer Exponents:**</u> Suppose a and b are nonzero real numbers and m and n are integers.

- For any positive integer n, we define $a^n =$ ________________, where a is the ________________ and n is the ________________.
- We also define $a^0 =$ ________.
- **Product Rules:**
  - ▷ $(ab)^n =$ ________________
  - ▷ $a^m \cdot a^n =$ ________________
- **Quotient Rules:**
  - ▷ $\left(\frac{a}{b}\right)^n =$ ________________
  - ▷ $\frac{a^m}{a^n} =$ ________________
- **Power Rule:** $(a^m)^n =$ ________________
- **Negative Exponents:**
  - ▷ $a^{-n} =$ ________________
  - ▷ $\frac{1}{a^{-n}} =$ ________________
  - ▷ $\frac{a^{-n}}{b^{-m}} =$ ________________
  - ▷ $\left(\frac{a}{b}\right)^{-n} =$ ________________

CAUTION: It is very important to note that you cannot distribute an exponent to an addition/subtraction. In general, $(a+b)^n \neq a^n + b^n$.

(EX) Perform the indicated operations and simplify.

(a) $3^{-1} - 4^{-2}$

(b) $\left(-\frac{3}{5}\right)^{-2} + 4^0$

(c) $(-2)^{-3} + (-3)^{-2}$

(d) $\dfrac{3 \cdot 5^{100}}{12 \cdot 5^{98}}$

(e) $(10^{11} \cdot 10^5)^3$

WeBWorK

Simplify

$$6^{-1} - 3^{-3}$$

and write your answer as a single fraction.

Answer: ______

(EX) Use rules of exponents to find the missing expression (marked with a ?).

(a) $\dfrac{1}{125} = 5^?$

(b) $4^{3x} = 2^?$

(c) $64x^8 = (?)^2$

(d) $-27n^{15} = (?)^3$

(e) $A^{2k} \cdot A^{3k+1} = A^?$

(f) $\dfrac{(Z^{5c})^2}{Z^{c+3}} = Z^?$

WeBWorK

Rewrite the following as a difference of cubes.

$$1000c^{27} - 27u^{21} = (\quad)^3 - (\quad)^3$$

Rewrite the following using a single exponent, and simplify. If

$$\frac{z^{3c}\left(z^{3c}\right)^2}{z^{3c+3}} = z^m$$

then $m =$ ______

Order of Operations

It is extremely important that we use the correct order of operations when simplifying and evaluating expressions. The order of operations that is expected is shown below:

P E M D A S

1. P________________________
2. E________________________
3. M________________________/ D________________________
4. A________________________/ S________________________

To get started, consider -5^2. For this, order of operations tells us that we must square the 5 before we make it negative. So, we really have:

$$-5^2 = -(5^2) = -25$$

If we had intended to square the -5, we would need to include it entirely in parentheses:

$$(-5)^2 = (-5)(-5) = 25$$

Now, consider a few more examples:

(EX) Simplify:

(a) $3 \cdot 2 - 4 \cdot 2^2 + 6(3-1)$

(b) $\dfrac{\frac{2}{3} - \frac{4}{5}}{4 - \frac{7}{10}}$

(c) $\dfrac{[4(8-6)^2 + 4](3 - 2 \cdot 8)}{2^2(2^3+5)}$

(d) $\dfrac{1 + 2^{-3}}{3 - 4^{-1}}$

WeBWorK

Use the order of operations to simplify:

(a) $5^2 - 27 \div 3^2 \cdot 5 - 9 =$ ______

(b) $\dfrac{6 \cdot 2 - 2^2}{\left[3^2 - (-3)\right]^2} =$ ______

Roots and Radicals: Taking a root of a real number is the way that we can 'undo' an exponent.

A number c is said to be an ______________ of a if ______________. We can represent a root using two approaches:

Radicals: $c = \sqrt[n]{a}$

Terminology: $n =$ ______________

$a =$ ______________________

Rational Exponents: $c = a^{1/n}$

Depending on whether the type of root you are finding is even or odd, there are certain properties to note:

- If n is ________:
  - An odd root can be found for any real number a (positive, negative, or zero).
  - The sign $(+/-)$ of the resulting root will be identical to the sign of the original value of a. (This matches with the fact that when you raise a number to an odd integer power, the result has the same sign as the original number.)
- If n is ________:
  - An even real root can be found for any non-negative real number a (positive, or zero). Although it is possible to find an even root of a negative number, the result is not a real number, and will not be covered in this course.
  - Any positive number a will have two real-number even roots. For example, there are two integers that can be squared to equal 9: (-3) and $(+3)$. We call the positive root the ______________________, and we define $\sqrt[n]{a} = a^{1/n}$ to be the principal root of a when n is even.
- The n^{th} root of $0 = \sqrt[n]{0} = 0^{1/n} = 0$ for any positive integer n (regardless of whether n is even or odd).

(EX) Write the root in the other form (either radical or rational exponent), and evaluate:

	Radical	Rational Exponent	Value
(a)	$\sqrt{25}$		
(b)		$81^{1/4}$	
(c)	$\sqrt[3]{-125}$		
(d)		$(-64)^{1/2}$	

We can extend our definition of the n^{th} root of a to describe the m^{th} power of the n^{th} root of a:

Radicals

$$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

Rational Exponents

$$\left(a^{1/n}\right)^m = (a^m)^{1/n} = a^{m/n}$$

Note that in this case, you can either apply the exponent or the root first, but when performing calculations by hand, it is usually easier to evaluate the root first.

(EX) Write the root in the other form (either radical or rational exponent), and evaluate:

	Radical	Rational Exponent	Value
(a)	$\sqrt{9^3}$		
(b)		$8^{4/3}$	
(c)	$\sqrt[3]{\left(\frac{1}{64}\right)^2}$		
(d)		$(-32)^{4/5}$	
(e)	$\sqrt[5]{32^{-2}}$		
(f)		$\left(\frac{16}{9}\right)^{-3/2}$	

Negative Exponents Produce Fractions

$$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

Rational Exponents Produce Roots

$$7^{1/2} = \sqrt{7}$$

WeBWorK

Simplify. Write **not real** if the answer is not a real number.

(a) $(-243)^{2/5} =$ ______

(b) $-25^{-5/2}$ ______

Simplifying Radicals: Many radical expressions cannot be fully simplified to an expression without any roots. In the problems below, we will look at how to simplify such radical expressions.

Properties of Radicals: Let a and b be any real numbers or expressions for which the given roots exist. For any natural numbers m and n $(n \neq 1)$:

- If n is even, $\sqrt[n]{a^n} =$ ________
- If n is odd, $\sqrt[n]{a^n} =$ ________
- Product Rule: $\sqrt[n]{ab} =$ ____________
- Quotient Rule: $\sqrt[n]{\frac{a}{b}} =$ ____________

(EX) Perform the indicated operations and simplify.

(a) $\sqrt[3]{54}$

(b) $\sqrt{75} - \sqrt{12}$

(c) $\sqrt[3]{\dfrac{16}{27}}$

(d) $\sqrt{3^2 + 4^2}$

(e) $\dfrac{-4 - \sqrt{(4)^2 - 4(3)(-2)}}{2(3)}$

(f) $(\sqrt{18} - \sqrt{8})^2$

(g) $(-8)^{2/3} - 9^{-3/2}$

(h) $(-64)^{1/3} + (-64)^{1/2}$

WeBWorK

Simplify.

(a) $\sqrt{8} - \sqrt{72}$

(b) $\dfrac{2\sqrt{2}}{3} + \dfrac{9\sqrt{2}}{5}$

(EX) Use rules of exponents to find the missing expression (marked with a ?).

(a) $\sqrt[5]{3} = 3^{?}$

(b) $\dfrac{1}{\sqrt[3]{10}} = (10)^{?}$

(c) $\sqrt[4]{A^3} \cdot \sqrt{A} = A^{?}$

WeBWorK

Simplify and write the following using rational exponents. If

$$\frac{\sqrt[9]{v^7}}{\sqrt[4]{p^7}} = v^m p^k,$$

then

$m =$ ______ and $k =$ ______

Simplify and write the following using a single rational exponent of z. If

$$\frac{\sqrt{z^3}\,\sqrt[4]{z^9}}{\sqrt[6]{z^7}} = z^m$$

then

$m =$ ______

0.3: Linear Equations and Inequalities

Solving Linear Equations: General Procedure

1. Eliminate fractions by multiplying all terms on both sides by the LCD.
2. Eliminate parentheses by distributing.
3. A linear equation in a variable X can be written in the form ________________, where $A, B \in \mathbb{R}$, $A \neq 0$. With linear equations you need to get all terms with the variable of interest on one side and factor to isolate the variable of interest.

(EX) Solve.

(a) $8x - (3x - 5) = 2(4 - 3x) - 7(1 - x)$

(b) $\dfrac{2(w-3)}{5} + 1 = \dfrac{4}{15} - \dfrac{3w+1}{9}$

(c) $7 - (4 - x) = \dfrac{2x-3}{2}$

WeBWorK

Solve the following equation for z:

$$\frac{5z+1}{5} - 5z = \frac{6z+7}{2} - 5$$

$z =$

Note: Write **no solutions** if no solutions exist or **infinitely many** if there are infinitely many solutions.

(EX) Solve the formula for the indicated variable.

(a) Solve for w: $vw - 1 = 3v$

(b) Solve for v: $vw - 1 = 3v$

(c) Solve for y: $x(y-3) = 2y + 1$

(d) Solve for b_1: $A = \frac{1}{2}h(b_1 + b_2)$

WeBWorK

Solve the following equation for z:

$$v(q+z) = qz$$

$z =$

Solving Linear Inequalities: General Procedure

1. Eliminate fractions by multiplying all terms on both sides by the LCD.
2. Eliminate parentheses by distributing.
3. Remember that if you ever multiply or divide both sides of the inequality by a negative number, the inequality sign(s) must be reversed.

(EX) Solve. Write your answer using interval notation.

(a) $1 - \frac{7-y}{4} \geq 3y + 1$

(c) $\frac{10m+1}{5} > 2m - \frac{1}{2}$

A compound inequality is the combination of two inequalities.

- If the two inequalities are joined by an 'and', then we must find the __________________________ of their solution sets.

  Ways of writing 'and' compound inequalities: $-2 \leq 3x + 1 \leq 4$; $4x - 1 \geq 2$ and $x + 2 < 7$

- If the two inequalities are joined by an 'or', then we must find the ________________ of their solution sets.

(EX) Solve. Write your answer using interval notation.

(a) $2x + 1 \leq -1$ or $2x + 1 \geq 1$

(b) $4 - x \geq 0$ or $2x + 7 < x$

(c) $2 \leq 3 - y < 7$

(d) $4 - x \geq 5$ and $4 - x \leq 3x$

Solve the following inequality. Write your answer in interval notation or if no solutions exist write **no solutions**.

$$4x + 3 > 15 - 2x \quad \text{or} \quad 4 - 3x > 13$$

Answer:

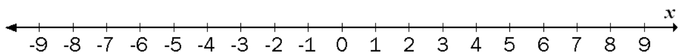
Solve the following inequality. Write your answer in interval notation or if no solutions exist write **no solutions**.

$$x - 4 \leq 20 - 2x \quad \text{and} \quad 4x + 4 > 4$$

Answer:

0.4: Absolute Value Equations and Inequalities

Absolute Value (distance from zero)



$$\text{For any real number } x,\ |x| = \begin{cases} \underline{\hspace{2cm}} & \text{if } \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \text{if } \underline{\hspace{2cm}} \end{cases}$$

Solving Absolute Value Equations:

1. First, isolate the absolute value expression on one side of the equation. (Write the equation in the form $|X| = c$, where c is a constant, and X is a quantity involving your variable.)

2. Second, determine how to solve based on the value of c:

Is c negative $(c < 0)$?	Is $c = 0$?	Is c positive $(c > 0)$?
Then there is <u>No Solution</u>.	Then solve $X = 0$.	Then solve BOTH $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$.

(EX) Solve:

(a) $|x + 1| = 4$

(b) $|3x + 5| - 2 = -1$

(c) $|2x - 5| + 8 = 3$

(d) $|2x + 1| = 0$

WeBWorK

Solve the equation for r:

$$24 - |r + 4| = 13$$

$r =$

Help: Enter your answers as a comma separated list if there is more than one correct answer. Type **no solution** if the equation has no solution.

Solving Absolute Value Inequalities:

1. First, isolate the absolute value expression on one side of the inequality.
2. Second, solve a compound inequality (either intersection or union, based on your original inequality.)

	First Inequality	AND/OR	Second Inequality
$\lvert X\rvert < c$			
$\lvert X\rvert \leq c$			
$\lvert X\rvert > c$			
$\lvert X\rvert \geq c$			

(EX) Solve. Write your answer using interval notation.

(a) $|5x+3| \leq 2$

(b) $|5x-11| < -7$

(c) $|2+\frac{3}{4}x| \geq -5$

(d) $|4-3x| > 1$

(e) $13 - |4 - 2x| < 10$

(f) $|2 + \frac{4}{3}x| > 0$

(g) $|5x - 11| \leq 0$

WeBWorK

Solve the following inequality. Write your answer using interval notation.

$$|6 - 10y| - 5 < 2$$

If there is no solution, write "No Solution" in the blank provided.

0.5: Polynomial Arithmetic

Polynomials

A polynomial in one variable is any expression of the type

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where n is the ________________ of the polynomial (a non-negative integer),

$a_0, a_1, \ldots, a_n$ are the __________________________ (real numbers),

a_n is the __________________________, and a_0 is the __________________________.

The polynomial is said to be in ____________________________________ if the exponents decrease from left to right.

We often describe polynomials based on the number of terms and the degree:

- one term = ________________
- two terms = ________________
- three terms = ________________

- degree 0 = ________________
- degree 1 = ________________
- degree 2 = ________________
- degree 3 = ________________

(EX) Find the degree of the polynomial:

(a) $9x^2 + x - 5x^5$

(b) 4

A polynomial in several variables is the term used to describe expressions like $ab^3 + 5ab - b^2$ or $8x^2y^5 - 1$. The degree of a term is the sum of the exponents of the variables in that term, and the degree of the polynomial is the largest degree of the individual terms.

(EX) Find the degree of the polynomial:

(a) $ab^3 + 5ab - b^2$

(b) $8x^2y^5 - 1$

Addition and Subtraction of Polynomials

Terms with the same variables raised to the same powers are called __________________________. These may be combined using addition or subtraction.

(EX) Perform the indicated operations and simplify:

(a) $(5x^2 + 4xy - 3y^2 + 2) + (9x^2 - 4xy + 2y^2 - 1)$

(b) $(5x^2 + 4xy - 3y^2 + 2) - (9x^2 - 4xy + 2y^2 - 1)$

Multiplication of Polynomials

(EX) Multiply and simplify:

(a) $(x+5)(2x-1)$

(b) $(2a-3b)(2a-b)$

(c) $(3x+y)(9x^2-3xy+y^2)$

WeBWorK

Perform the operation

$$(2z-6)(4z-8)-(-6z-2)(-9z-3)$$

and combine line terms. Simplify your answer as much as possible.

Answer:

Special Products of Binomials

- $(A+B)^2 =$ ____________________
- $(A+B)(A-B) =$ ____________________
- $(A-B)^2 =$ ____________________

(EX) Perform the indicated operations and simplify:

(a) $(3x-2)^2$

(b) $(4+3x)(4-3x)$

(c) $2x(4+x)^2$

(d) $(x-5y-1)(x-5y+1)$

(e) $[2(x+h)-(x+h)^2]-(2x-x^2)$

WeBWorK

Perform the operation

$$(5v-1)^2-(5v+1)^2$$

and combine like terms. Simplify your answer as much as possible.

Answer:

Polynomial Long Division: Dividing two polynomials follows the same general process used when dividing natural numbers, so let's review that first:

dividend = (divisor)(quotient) + remainder

When working with polynomials, we can perform long division if the degree of the divisor is less than or equal to the degree of the dividend. We'll know that we're done when the degree of the polynomial that is left over (the remainder) is smaller than the degree of the divisor.

(EX) Use long division to find the quotient and remainder. Check your answer by showing that *dividend = (divisor)(quotient) + remainder*.

(a) $\dfrac{x^3 + 4x^2 - 6x + 1}{x + 1}$

(b) $\dfrac{3x^3 - 2x + 1}{x - 2}$

(c) $(5t^3 - t + 1) \div (t^2 + 4)$

WeBWorK

Given the polynomial $P = -3x^4 - 6x^3 + 7x^2 - 4$,

and divisor $d = -x^2 - 2$,

find the quotient Q and remainder R.

Write your answer in the form $P = d \cdot Q + R$

$-3x^4 - 6x^3 + 7x^2 - 4 = (-x^2 - 2)(\quad\quad) + (\quad\quad)$

0.6: Factoring

Factoring is an algebraic skill that will be critical to completing many of the upcoming problems in this course. It is important that you understand basic factoring techniques, and that you master them to the point where you can factor without having to stop and think for a long time about the problem. Note that when you are asked to factor a problem in this course, you will be "factoring over the integers". This means that we will not have answers with expressions like $(x+\sqrt{3})$ or $(x-\frac{1}{3})$. Instead, we will restrict our answers to integer coefficients only.

FACTORING: Greatest Common Factors

When we factor expressions, the first thing we should always look for is a greatest common factor (GCF). The greatest common factor is the largest factor that divides evenly into each term of the expression. For example:

(EX) Factor by pulling out the greatest common factor:

(a) $4x^3+8x-2$

(b) $18a^3b^2+6ab^4-9a^2b^3$

NOTE: You can always check your answer from factoring by multiplying the expression back together. The result should equal the original expression.

After checking for a GCF, we can break down our remaining factoring techniques by the number of terms in the expression that needs to be factored.

TWO TERMS: Special Factoring Formulas

Below are three special factoring formulas that you will be expected to use during this course.

The Difference of Squares: $A^2-B^2 = \underline{\hspace{5cm}}$

The Sum of Squares: $A^2+B^2 = \underline{\hspace{5cm}}$

The Difference of Cubes: $A^3-B^3 = \underline{\hspace{5cm}}$

The Sum of Cubes: $A^3+B^3 = \underline{\hspace{5cm}}$

Note that there are two other special formulas that you do not need to memorize (since you can factor these expressions using methods we will look at later in this section). But, in the interest of completeness, they are included below:

Perfect Square Binomials: $A^2-2AB+B^2 = \underline{\hspace{3cm}}$

$A^2+2AB+B^2 = \underline{\hspace{3cm}}$

(EX) Factor completely over the integers:

(a) $4x^2-81$

(b) $4x^2+81$

(c) $18y-2x^2y$

(d) $8x^3+1$

WeBWorK

Rewrite the expression by taking out the greatest common factor and putting it in front.

$12x^{13} + 48x^{10} + 12x^{9} + 16x^{7} =$ ______ (______)

Help: You do not need to factor the remaining expression after factoring out the greatest common factor.

Factor the expression and simplify your answer as much as possible:

$144q^4 - 25x^2 =$ ______

$27x^3 - 125 =$ ______

Help: Make sure to put parenthesis around each individual factor!

FOUR TERMS: Factoring by Grouping

If the expression that you are factoring contains four terms, then factoring by grouping is the best approach to try first. In this technique, we think about the different terms as being members of smaller groups. We could group the four terms into groups of one/three, three/one, or two/two. For this course, we will focus on the idea of two/two grouping.

(EX) Factor the expression by grouping:

(a) $t^3 - 4t^2 + 7t - 28$

(b) $6x^3 + 12x^2 + x + 2$

(c) $5x^3 + 3x^2 - 30x - 18$

(d) $2a^3 - 5a^2 - 18a + 45$

WeBWorK

Factor the expression and simplify your answer as much as possible:

$2x^3 - x^2 - 8x + 4 =$ ______

Help: Make sure to put parenthesis around each individual factor!

THREE TERMS: Factoring Trinomials $ax^2 + bx + c$

When we encounter an expression with three terms (in the form $ax^2 + bx + c$), we expect that *if* it factors, it will generally factor into the product of two binomials. (*You should note that not every trinomial will factor.*) There are two techniques that can be used to factor a trinomial into its binomial factors.

Technique #1: Reverse FOIL Method / Guess & Check

For the first factoring technique, we use a guess & check approach (or trial & error) to try to determine what the factors will be. Since the approach is based on the idea of formulating a "good guess", it is sometimes labor intensive. However, students who practice and master this technique typically become quite efficient at factoring using this approach.

Factor: $3x^2 - x - 2 = (\boxed{?}x + \boxed{?})(\boxed{?}x + \boxed{?})$

1. Begin by finding the factors of 3 (the coefficient of the First term) and -2 (the Last term):

   Factors of 3: ________________ Factors of -2: ________________

2. Pick any set of factors and substitute them in for your guess, in the First and Last positions, respectively. Then check to see if the Outer + Inner adds up to the middle term. Note that you may want to reverse the order of your factors sometimes to try out different combinations.

3. Often, this technique works well when either the First or the Last constant in the trinomial is a prime number. This will limit the number of combinations that may need to be tried.

NOTE: When you examine the signs (+/-) of the terms of your trinomial, it can give you some information about the signs (+/-) that will be contained within your factored result.

$ax^2 + bx + c$	$ax^2 - bx + c$	$ax^2 + bx - c$	$ax^2 - bx - c$
factors into the form	factors into the form	factors into the form	factors into the form
(Px ___ Q)(Rx ___ S)	(Px ___ Q)(Rx ___ S)	(Px ___ Q)(Rx ___ S)	(Px ___ Q)(Rx ___ S)

(EX) Factor completely over the integers:

(a) $6x^2 - 13x - 5$

(b) $12x^2 + 8x - 15$

Technique #2: ac-Method / Grouping Method

The second technique that can be used to factor a trinomial into a product of two binomials is called the ac- or Grouping Method. This technique is more systematic, but there still may be some difficulty in finding the correct combination of factors if the coefficients are particularly large.

STEP 1. Factor out the GCF, if any.

STEP 2. Multiply $a \cdot c$.

STEP 3. Find two numbers whose product $= a \cdot c$, AND sum $= b$.

STEP 4. Write the bx term as a sum using the numbers found in Step 3.

STEP 5. Factor by grouping.

Factor: $3x^2 - x - 2$

(EX) Factor completely over the integers:

(a) $6x^2 - 13x - 5$

(b) $8w^2 - 42w - 11$

WeBWorK

Factor the expression and simplify your answer as much as possible:

$$6b^2 + 7b + 2 = \underline{\qquad\qquad}$$

Help: Make sure to put parenthesis around each individual factor!

Rewrite the expression by taking out the greatest common factor and putting it in front. Then factor the remaining expression as much as possible and type your result in the second answer blank.

$$18v^3 + 15v^2 - 18v = \underline{\qquad}\ \underline{\qquad\qquad}$$

Help: Make sure to put parenthesis around each individual factor!

THREE TERMS: Trinomials With Two Variables

You may be asked to factor an expression like $r^2 - 5rs - 6s^2$, which is a trinomial with two variables. Notice that the degree of r decreases as you read from left to right, while the degree of s increases as you read from left to right. Often, these trinomials will factor in a way that is similar to the approach outlined above, except the resulting binomials will have two variables instead of only one.

(EX) Factor completely over the integers:

(a) $r^2 - 5rs - 6s^2$

(b) $10m^2 + 7mn - 12n^2$

Factoring Using Substitution

There are times when the expression you wish to factor is not exactly in the format we have already discussed, but it may be similar. In these cases, we can use a substitution to help aid in factoring.

(EX) Factor completely over the integers:

(a) $2(y + 4)^2 + 7(y + 4) + 6$

(b) $x^4 - 3x^2 - 40$

(c) $(2x + 3)^2 - 9$

WeBWorK

Factor the expression and simplify your answer as much as possible:

$$100y^2 + 180yz + 81z^2 =$$

$$t^6 - 2t^3 - 15 =$$

$$(s-3)^3 + 125 =$$

Help: Make sure to put parenthesis around each individual factor!

SUMMARY: Factoring Techniques

Here is a basic plan that can help you to tackle most of the factoring problems you will encounter in this course:

1. Factor out the GCF, if any.
2. How many terms do you have?
   (a) For TWO terms: Try a special factoring formula for $A^2 - B^2$, $A^3 - B^3$, or $A^3 + B^3$.
   (b) For THREE terms: Try the ac-method, or the Guess & Check method.
   (c) For FOUR terms: Try factoring by grouping.
3. Always check to see if any of your factors can be broken down further.
   (For example, $(x^2 - 9)(x + 2)$ can be factored further to $(x - 3)(x + 3)(x + 2)$.)
4. Consider whether a substitution may help you to factor.
5. Remember that you can multiply your factors back together to check whether or not your factorization is correct.

Solving Equations by Factoring: Factoring is our main tool for solving non-linear equations in this course.

Strategy for Solving Non-linear Equations:

1. Clear fractions (if appropriate) by multiplying both sides of the equation by the LCD.
2. Distribute and clear all parentheses.
3. Put all of the non-zero terms on one side of the equation so that the other side is 0.
4. Factor.
5. Use the Zero Product Property ($ab = 0$ if and only if $a = 0$ or $b = 0$) and set each factor equal to 0.
6. Solve each of the resulting equations.

(EX) Solve:

(a) $4t = t^2$

(b) $w(6w + 11) = 10$

(c) $2w^2 + 5w + 2 = -3(2w + 1)$

(d) $\dfrac{x^3 - x}{2} = \dfrac{x^2 - 1}{3}$

WeBWorK

Solve the equation for t:

$$(t + 5)(t + 2) = 4$$

$t =$

Help: Separate multiple answers by a comma separated list.

0.7: Quadratic Equations

Quadratic Equations: A quadratic equation is an equation of the form ______________________________, where $a, b, c \in \mathbb{R}$, and $a \neq 0$. We already know how to solve quadratic equations which can be factored, but we will now learn additional techniques that can be used even when the quadratic equation cannot be factored.

Solving Quadratic Equations by Extracting Square Roots: (Use when you are solving for a variable contained within a perfect square.)

If c is a real number with $c \geq 0$, then the solutions to $X^2 = c$ are ________________ or ________________.
If $c < 0$, then $X^2 = c$ has no real number solutions.

(EX) Solve by extracting square roots.

(a) $x^2 + 4 = 13$

(b) $(y-3)^2 = 18$

(c) $(1-2t)^2 = 9$

(d) $4(x+2)^2 - 32 = 0$

WeBWorK

Solve the equation for s:

$$4s^2 - 52 = 0$$

$s =$

Perfect Square Trinomials

Recall from our work with polynomials that a perfect square trinomial always has one of two forms:

$$(A^2 + 2AB + B^2) = (A+B)^2$$
$$(A^2 - 2AB + B^2) = (A-B)^2$$

Specifically, here are a few examples:

$$x^2 + 8x + 16 = (x+4)^2 \qquad x^2 - 10x + 25 = (x-5)^2 \qquad x^2 + 12x + 36 = (x+6)^2$$

What is the relationship between b (the coefficient of x) and c (the constant term)?

(EX) Find the constant that should be added to each expression in order to create a perfect square trinomial. Then, factor the result.

(a) $a^2 - 10a$

(b) $p^2 + 9p$

Completing the Square

The method of completing the square is a technique that allows us to rewrite a quadratic equation $ax^2+bx+c=0$ in the form $a(x-h)^2+k=0$, so that we can extract square roots to solve.

Completing the square for $\mathbf{ax^2+bx+c=0}$:

STEP 1. Subtract c from both sides.

STEP 2. Divide both sides by a (the coefficient of x^2).

STEP 3. Multiply the coefficient of x by $\frac{1}{2}$, and square the result. Add this value to both sides of the equation.

STEP 4. Factor the left hand side of the equation as a perfect square.

STEP 5. Extract square roots and solve for x.

(eg) $2x^2 - 12x + 5 = 0$

(EX) Solve by completing the square.

(a) $x^2 - 2x - 17 = 0$

(b) $3y^2 + 12y + 5 = 0$

WeBWorK

Follow the steps to solve the equation:

$$x^2 - 7x + 7 = 0$$

by completing the square.

Step 1: First move c over and then add $\left(\frac{b}{2}\right)^2$ to both sides.

$$x^2 - 7x + \underline{\qquad} = -7 + \underline{\qquad}$$

Step 2: Now rewrite the LHS as $\left(x + \frac{b}{2}\right)^2$ and simplify the RHS.

$$(x + \underline{\qquad})^2 = \underline{\qquad}$$

Step 3: Use the square root property to solve for x.

$x = \underline{\qquad}$

The Quadratic Formula:

If we use the technique of completing the square on the equation $ax^2 + bx + c = 0$, we can develop a formula based on a, b, and c which provides the solutions to the equation.

The Quadratic Formula: $x = \underline{\qquad\qquad\qquad}$

(EX) Solve using the quadratic formula:

(a) $3n^2 - 8n + 3 = 0$

(b) $\dfrac{1}{4}m^2 + \dfrac{1}{3} = -\dfrac{m}{2}$

(c) $12v^2 + v - 1 = 5v^2 + 5v$

WeBWorK

Solve the equation for x:

$$\left(x - \frac{1}{4}\right)^2 = \frac{x}{4}$$

$x =$ ______

Discriminant:

Given a quadratic equation $ax^2 + bx + c = 0$, the ______________________________ is defined to be:

$$D = \underline{\hspace{4cm}}$$

- If $D > 0$, then there are __________ distinct real number solutions to the equation.
- If $D = 0$, then there is __________ repeated real number solution.
- If $D < 0$, then there are __________real solutions.

Strategies for Solving Quadratic Equations

1. If the variable appears in the squared term only, isolate it and extract square roots.
2. Otherwise, put the nonzero terms on one side of the equation so that the other side is 0.
   (a) Try factoring, and solve using the Zero Product Property.
   (b) If the expression doesn't factor easily, use the Quadratic Formula.
   (c) As a third option, you can Complete the Square.

Identifying Quadratics in Disguise:

An equation is a 'Quadratic in Disguise' if it can be written in the form:

$$aX^{2m} + bX^m + c = 0$$

In other words,

- There are exactly three terms: two with variables, and one constant term.
- The exponent on the variable in one term is *exactly* ________________ the variable on the other term.

To transform a Quadratic in Disguise into a quadratic equation, use the substitution $u = X^m$.

(EX) Solve:

(a) $2x^4 - 9x^2 = -4$

(b) $2(t+2)^2 - 5(t+2) - 12 = 0$

WeBWorK

Solve the equation for s:

$$(s+3)^2 - 2(s+3) - 8 = 0$$

$s =$

0.8: Rational Expressions and Equations

Simplifying, Multiplying, and Dividing Rational Expressions (including Canceling)

$\dfrac{AC}{BC} =$ ______________________

$\dfrac{A}{B} \cdot \dfrac{C}{D} =$ ______________________

$\dfrac{A}{B} \div \dfrac{C}{D} =$ ______________________

$\dfrac{A}{B} \cdot \dfrac{B}{C} =$ ______________________

The Opposite of a Binomial

Recall that the opposite of a number (a) is defined as $(-a)$.

Similarly, the opposite of an algebraic expression $(b-a)$ is defined to be $-(b-a) = a-b$. If we rewrite in descending order, and factor out the negative sign, we have:

$$-(b-a) = -(-a+b) = a-b$$

When simplifying algebraic expressions, we may encounter situations where we have opposites that can be reduced or replaced. For example:

$\dfrac{6}{3-x} =$

$\dfrac{3-x}{x-3} =$

WeBWorK

Which of the following expressions are equal to 1, -1, or to neither of those? List the corresponding letter(s), separated by commas if there are more than one.

A. $\dfrac{a-14}{-a+14}$ B. $\dfrac{a+14}{14+a}$ C. $\dfrac{-14+a}{a-14}$

D. $\dfrac{a-14}{14-a}$ E. $\dfrac{a+14}{a-14}$ F. $\dfrac{-a-14}{a+14}$

Equals 1:

Equals -1:

Neither:

(EX) Simplify:

(a) $\dfrac{y^5 - 5y^4 + 4y^3}{y^3 - 6y^2 + 8y}$

(b) $\dfrac{a^2-b^2}{x-y}\cdot\dfrac{y-x}{a-b}$

(c) $\dfrac{8x^3-1}{6x^2+7x-5}\div\dfrac{8x^2-2}{9x^2-25}$

WeBWorK

Perform the indicated operation and simplify if possible. Assume all cancelled factors are not zero.

$$\frac{12b-36}{5a+10}\cdot\frac{a^2-12a+36}{ab-3a-6b+18}=$$

Adding and Subtracting Rational Expressions

$\dfrac{A}{B}+\dfrac{C}{B}=$ ____________________

$\dfrac{A}{B}+\dfrac{C}{D}=$ ____________________

(EX) Find the LCD of the rational expressions:

(a) $\dfrac{4}{b},\dfrac{5}{b+3}$

(b) $\dfrac{y+1}{y^2-6y+8},\dfrac{3}{y^2-16}$

(EX) Simplify:

(a) $\dfrac{a}{a^2+10a+25}-\dfrac{4-a}{a^2+6a+5}$

(b) $3w^{-2}-(3w)^{-1}$

(c) $\dfrac{9x-12}{x^2-x-6} - \dfrac{7}{x+2} + \dfrac{3}{3-x}$

(d) $\left(\dfrac{3x}{x+2} - \dfrac{2+3x}{x-2}\right) \cdot \left(\dfrac{x^3-8}{7x+2}\right)$

WeBWorK

Perform the indicated operation, and simplify if possible. Assume any factors you cancel are not zero.

$$\frac{4}{b^2-16} - \frac{b}{b^2+8b+16} + \frac{1}{b-4} =$$

Simplifying Complex Fractions

1. Combine the terms in the numerator and denominator to have a single fraction in each position.

2. Multiply the numerator by the reciprocal of the denominator.

3. Simplify.

Simplify: $\dfrac{\frac{a}{b} - \frac{b}{a}}{\frac{1}{a} - \frac{1}{b}}$

(EX) Simplify:

(a) $\dfrac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^2}-\frac{1}{b^2}}$

(b) $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}}$

(c) $\dfrac{x^{-1}+y^{-1}}{(x+y)^{-1}}$

WeBWorK

Perform the indicated operation, and simplify if possible. Assume any factors you cancel are not zero.

$$\frac{r^{-1}-8}{8+r^{-1}} = \frac{\square}{\square}$$

Expressions VS Equations:

It is important to understand the difference between an expression (which can be *simplified*), and an equation (which can be *solved*). Identify which of the following are expressions, and which are equations:

(A) $\dfrac{5x-9}{x}-\dfrac{4x}{x+1}$ (B) $\dfrac{5x-9}{x}=\dfrac{4x}{x+1}$ (C) $x^{-2}-9=0$ (D) x^2-9

Rational Equations:

1. Factor all denominators and find the LCD.

2. Clear fractions by multiplying every term in the equation by the LCD. (Note that you can ONLY clear fractions with an *equation*, and not an *expression*!)

3. Solve the remaining equation.

4. Check for extraneous solutions.

Solve: $\dfrac{x}{x+2} + \dfrac{5}{x} = \dfrac{4}{x^2+2x}$

(EX) Solve:

(a) $\dfrac{2}{x} = \dfrac{5}{x+1}$

(b) $\dfrac{2x+17}{x+1} = x+5$

(c) $\dfrac{x}{x-1} - \dfrac{1}{x+1} = \dfrac{2}{x^2-1}$

(d) $\dfrac{1}{t-4} + 6 = \dfrac{4}{t^2-4t}$

WeBWorK

Solve the equation for p:

$$\frac{p^2+20}{p^2-p-20}+\frac{p}{5-p}=\frac{p-5}{p+4}$$

$p =$

(EX) Solve for the indicated variable:

(a) Solve for y: $\dfrac{1}{x}+\dfrac{x}{y}=\dfrac{2}{z}$

(b) Solve for y: $x=\dfrac{1-2y}{y+3}$

WeBWorK

Solve the equation for R:

$$Z=1-\frac{P}{R}$$

$R =$

0.9: Radicals and Equations

Simplifying Radicals: Many radical expressions cannot be fully simplified to an expression without any roots. In the problems below, we will look at how to simplify such radical expressions.

Properties of Radicals: Let a and b be any real numbers or expressions for which the given roots exist. For any natural numbers m and n $(n \neq 1)$:

- If n is even, $\sqrt[n]{a^n} =$ _________
- If n is odd, $\sqrt[n]{a^n} =$ _________
- Product Rule: $\sqrt[n]{ab} =$ _________________
- Quotient Rule: $\sqrt[n]{\dfrac{a}{b}} =$ _________________

(EX) Simplify. Assume that all variables represent positive real numbers.

(a) $\sqrt[3]{-125x^6}$

(b) $\sqrt[5]{x^{23}}$

(c) $\sqrt[7]{y^{32}z^{49}}$

(d) $\sqrt{20s^3t^5}$

(e) $\sqrt{\dfrac{x^4y^3}{9}}$

(f) $\sqrt[5]{\dfrac{32n^{15}}{m^8}}$

(g) $\sqrt[3]{6x^2}\,\sqrt[3]{4x^2}$

(h) $\dfrac{\sqrt[3]{5ab^2}}{\sqrt[3]{40a^5b^5}}$

WeBWorK

Simplify completely assuming all expressions under radicals represent nonnegative numbers. If

$$\sqrt[4]{162z^{19}w^{30}} = A\sqrt[4]{B}$$

then

$A =$ ________ , $B =$ ________ .

Hint: It is possible that A and/or B could equal 1.

WeBWorK

Simplify. Assume that all variables represent positive quantities.

$$\sqrt{3x^4v^3}\sqrt{15x^7v^{11}} = $$

Write your answer using radical notation if necessary.

(EX) Simplify. Assume that all variables represent positive real numbers.

(a) $\sqrt[5]{(a+3)^3}\sqrt[5]{(a+3)^6}$

(b) $(\sqrt{10}-\sqrt{2w})^2$

(c) $(4x+\sqrt{7})(4x-\sqrt{7})$

WeBWorK

Simplify. Assume that all expressions under radicals represent nonnegative numbers.

$$\left(5\sqrt{c}-3\right)\left(3\sqrt{c}-5\right) = $$

Write your answer using radical notation if necessary.

Rationalizing Denominators: When we remove all radicals in the denominator of a rational expression, we call this process rationalizing the denominator. (Numerators can also be rationalized, using the same ideas.)

To rationalize the denominator, we multiply the top and bottom of the fraction by the ________________ of the denominator.

$$\frac{5}{\sqrt{2}+3}$$

(EX) Rationalize the denominator:

(a) $\dfrac{\sqrt{2}}{2-\sqrt{6}}$

(b) $\dfrac{\sqrt{q}}{1+\sqrt{q}}$

WeBWorK

Rationalize the denominator.

$$\frac{1-\sqrt{5}}{5\sqrt{7}-\sqrt{35}} =$$

Type an exact answer using radical notation if necessary.

Solving Power and Radical Equations:

We can generalize the ideas used in section 0.7 for extracting square roots to solve equations involving higher powers. Also, a similar idea can be used to solve equations involving roots.

Solving Power Equations	Solving Radical Equations
First, isolate the expression raised to a power.	First, isolate the radical.
$X^n = c$	$\sqrt[n]{X} = c$
Solve by taking the nth root of both sides.	Solve by raising both sides to the nth power.
If n is even, then the equation will only have a solution if $c \geq 0$, and $X = \pm\sqrt[n]{c}$.	If n is even, be sure to check for extraneous solutions.

(EX) Solve.

(a) $(2x+1)^3 = -8$

(b) $\sqrt{3x-1} = 4$

(c) $\dfrac{(1-2y)^4}{3} = 27$

(d) $5 - \sqrt[3]{t^2+1} = 1$

(e) $\sqrt{2x+5}+5=x$

(f) $\sqrt{5x+1}=x-1$

(g) $\dfrac{1}{1+2t^3}=4$

(h) $b^{2/3}=64$

WeBWorK

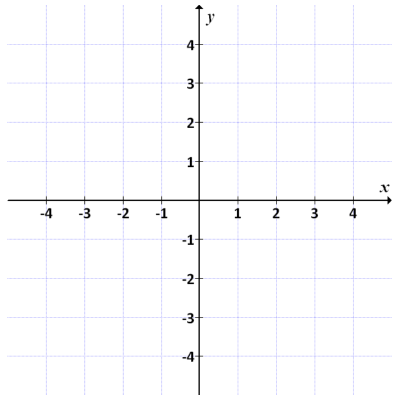
Solve the equation for u:

$$-2=-u+\sqrt{12u-44}$$

$u =$

1.1: The Cartesian Coordinate Plane

The Cartesian Plane



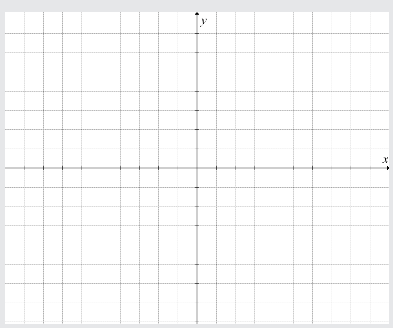
Graphs: To generate the graph of an equation, we are interested in finding all ordered pairs (x, y) such that x and y satisfy the equation.

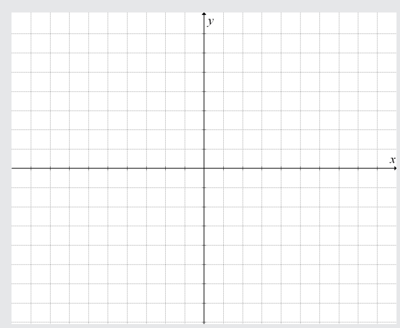
At this point, we don't have any specialized approaches for generating graphs. So, the most efficient approach for now is to generate a list of points that can be plotted, and 'connecting the dots'.

(EX) Graph the equation by using a table of values.

(a) $x = 2y + 3$

(b) $y = \sqrt{x} - 2$





WeBWorK

If (x, y) is a point in quadrant III, in what quadrant is the point $(-x, -y)$?

Quadrant:

If (x, y) is a point in quadrant III, in what quadrant is the point $(y, -x)$?

Quadrant:

Symmetry and Reflections: Consider the point $P(a, b)$ in the xy-plane.

- The point ________________ is symmetric with P about the x-axis.
- The point ________________ is symmetric with P about the y-axis.
- The point ________________ is symmetric with P about the origin.

We can also think of finding symmetric points as the process of reflecting a point about an axis or the origin.

To reflect a point (x, y) about the:

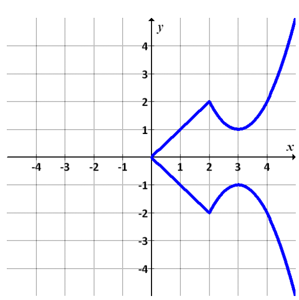
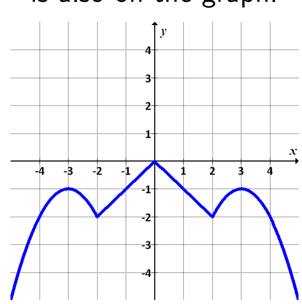
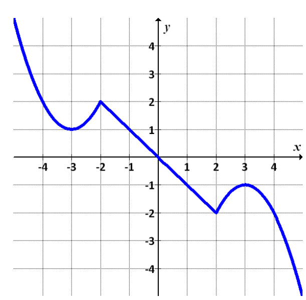
- x-axis, replace y with ________.
- y-axis, replace x with ________.
- origin, replace x with ________ and y with ________.

(EX) Complete the table below.

Point	Quadrant or Axis Where Point Lies	Reflection about the x-axis	Reflection about the y-axis	Reflection about the origin
$(-7, 10)$				
$(5, -3)$				
$(0, -2)$				

Symmetry of Graphs

A graph is symmetric with respect to the:

x-axis	y-axis	origin
if, for any point (x, y) on the graph, the point		
________________	________________	________________
is also on the graph.		
		

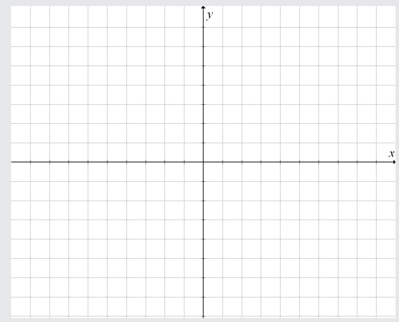
To test for symmetry algebraically:

If replacing y with $-y$ results in an equivalent equation, then the graph is symmetric with respect to the x-axis.	If replacing x with $-x$ results in an equivalent equation, then the graph is symmetric with respect to the y-axis.	If replacing both x with $-x$ and y with $-y$ results in an equivalent equation, then the graph is symmetric with respect to the origin.

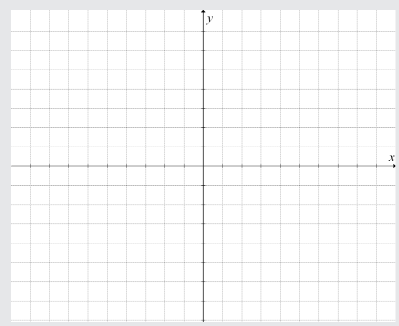
Question: Is it possible for an equation to have two types of symmetry? Three?

(EX) Determine whether the graph of each equation is symmetric about the x-axis, y-axis, or the origin. Sketch the graph.

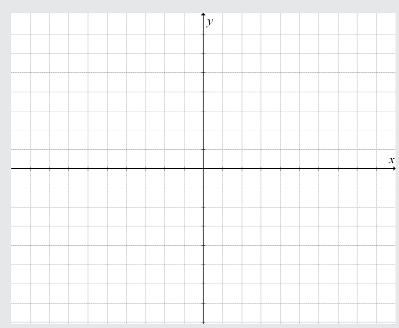
(a) $y = |x| + 1$



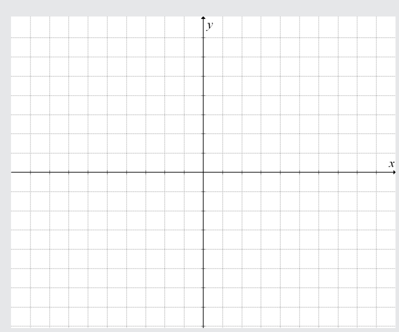
(b) $x^2 + y^2 = 9$



(c) $x = 2 - y^2$



(d) $y = x + 1$



WeBWorK

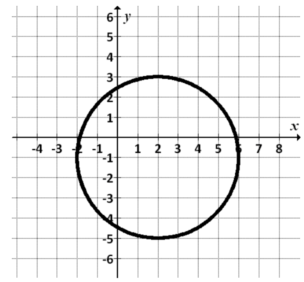
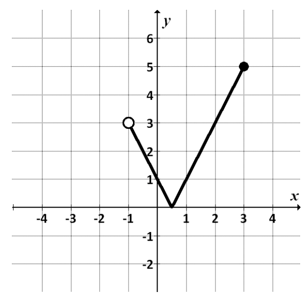
Determine the symmetries (if any) of the graphs of the given relations.

(a) $xy = 4$: select

(b) $6y = 5x^2 - 5$: select

1.3: Introduction to Functions

A ________________ is a correspondence between a first set, called the ________________, and a second set, called the ________________. If the correspondence has the property that each member of the domain corresponds to *exactly one* member of the range, then the relation is called a ________________.

peas grapes bananas milk chicken asparagus ham meat fruit vegetables dairy	$\{(2,5),(3,-6),(-2,12),(4,3)\}$	
meat fruit vegetables dairy peas grapes bananas milk chicken asparagus ham	$\{(2,5),(3,-6),(2,12),(3,4)\}$	

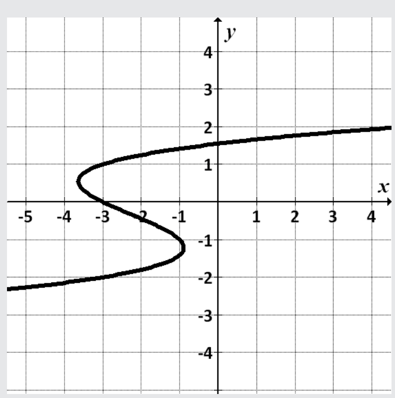
Vertical Line Test: If it is possible for a vertical line to cross a graph more than once, then the graph is NOT the graph of a function.

(EX) Determine whether the relation is a function. State its domain and range.

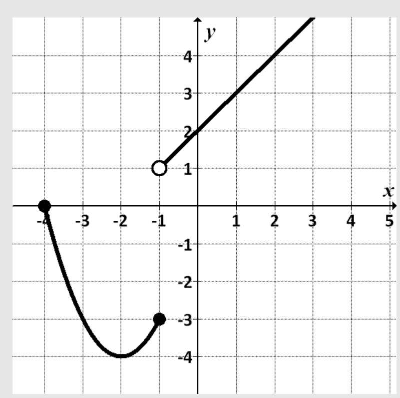
(a) $\{(2,3),(3,4),(4,3),(5,4)\}$

(b) $\{(5,-1),(-1,2),(2,7),(5,3)\}$

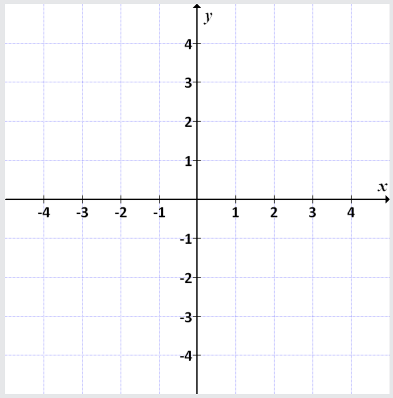
(c)



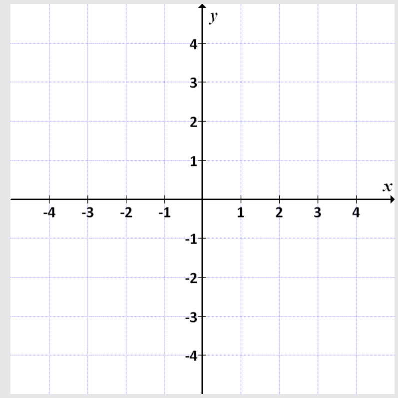
(d)



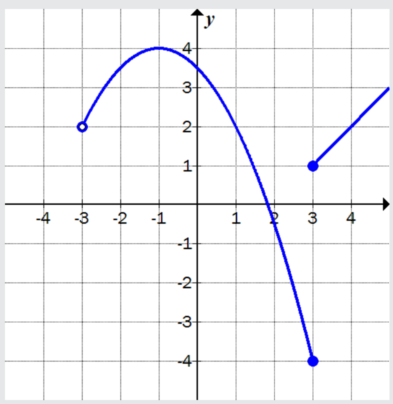
(e) $\{(1, y) \mid -3 < y \leq 2\}$



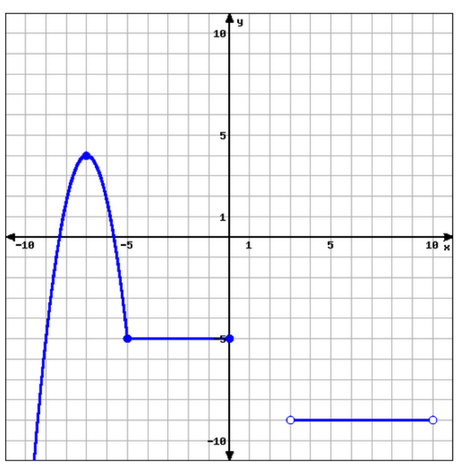
(f) $\{(x, -3) \mid x \geq 1\}$



(g)



WeBWorK



Given the graph of $f(x)$ above, find the following and write your answer in interval notation:

(a) Domain of $f(x)$:

(b) Range of $f(x)$:

1.4: Function Notation

Function Notation: We use the notation ________________ to denote a function. This is read "f of x", and does not indicate a multiplication.

For example, if $f(x) = -x^2 + 5x$, then:

- $f(1) =$ ________________
- $f(-3) =$ ________________
- $f(2) =$ ________________
- $f(t) =$ ________________
- $f(p^3) =$ ________________
- $f(a + 2b) =$ ________________
- $f(x - 2) =$ ________________
- $f(x) - 2 =$ ________________
- $f(x) - f(2) =$ ________________
- $f(3x) =$ ________________
- $3f(x) =$ ________________

(EX) Given $a(x) = \dfrac{x}{2 - x}$, and $b(x) = \dfrac{1}{\sqrt{x}}$, find:

(a) $a(3)$

(b) $b(75)$

(c) $a(-x)$

(d) $b(t + 1)$

(e) $a(x + h)$

(f) $b(-4)$

(g) $a(2)$

(h) $b(-v)$

(EX) If $f(x) = \sqrt{x-4}$, then $f(x^2+8) =$

① $\sqrt{x^2+4}$

② $x+4$

③ $x+2$

④ $x+6$

⑤ None of the above

WeBWorK

If $f(x) = x^2 + 2x - 2$, find the following.

(a) $f(x+a) =$

(b) $f(a) =$

(c) $f(x+a) - f(a) =$

(EX) Use the given function f to find $f(0)$ and solve $f(x) = 0$.

(a) $f(x) = x^3 + 3x^2 - 4x - 12$

(b) $f(x) = \dfrac{2}{x+5}$

WeBWorK

Given that $f(x) = \dfrac{-4}{x+2}$, find $f(0)$ and solve $f(x) = 0$.

(a) $f(0) =$

(b) $f(x) = 0$ whenever $x =$

<u>Domain of a Function:</u> The domain of a function is the set of all possible input values (usually this is the set of all x-values). As we work with functions, there are two main things we want to be mindful of for now:

1. **Do not divide by 0!**

> <u>PRINCIPLE FOR FINDING DOMAIN:</u>
>
> Domain of $f(x) = \dfrac{\text{numerator}}{\text{denominator}}$: ______________________________

2. **Do not take even roots of negative numbers!** (Although this is possible to do using imaginary numbers, we will not cover them in this course.)

> <u>PRINCIPLE FOR FINDING DOMAIN:</u>
>
> Domain of $f(x) = \sqrt[n]{\text{radicand}}$ (n is even): ______________________________

Some functions may have combinations of variables in the denominator and under even radicals, which will mean that we may need to apply more than one of these principles. If a function is a polynomial, neither of the rules above will apply:

> Domain of $f(x) =$ polynomial: ______________________

(EX) Find the domain of the function (in interval notation):

(a) $f(x) = \dfrac{x}{2-x}$

(b) $g(x) = \sqrt{4+x}$

(c) $h(x) = \dfrac{\sqrt{x}}{x^2-4x-12}$

(d) $k(x) = \dfrac{x^2-9}{\sqrt{3-6x}}$

(e) $j(x) = \dfrac{\sqrt[3]{3x}}{2+\frac{3x}{x-1}}$

(f) $m(x) = \dfrac{7}{5-\sqrt{x+2}}$

WeBWorK

Find the domain of the function

$$q(v) = \frac{3}{25 - \frac{1}{v^2}}$$

and write your answer using interval notation.

Domain: ______

Piecewise-Defined Functions: A function is defined piecewise, if it has different formulas for different parts of the domain.

(eg) $f(x) = \begin{cases} 2x - 1 & \text{for} \quad x < 1 \\ 3 - x & \text{for} \quad x \geq 1 \end{cases}$

$f(-3) =$ ______ $f(1) =$ ______ $f(4) =$ ______

(EX) Find the indicated values for the following function: $g(x) = \begin{cases} \frac{1}{2}x - 1 & \text{for} \quad x < 0 \\ 3 & \text{for} \quad 0 \leq x \leq 1 \\ -2x & \text{for} \quad x > 1 \end{cases}$

(a) $g(-2)$

(b) $g(0)$

(c) $g(1)$

(d) $g(1) - g(0)$

WeBWorK

Find the following values of the function

$$f(x) = \begin{cases} 4x^{-1} & x \leq 3 \\ x^2 + 6 & 3 < x \leq 8 \\ 9 + x & x > 8 \end{cases}$$

$f(3) =$ ______

$f(6) =$ ______

$f(10) =$ ______

1.5: Function Arithmetic

Sums, Differences, Products, and Quotients: If we have a function $f(x)$ with domain A, and $g(x)$ with domain B, then we define the following:

$(f+g)(x) =$ ______________________ Domain: ______________

$(f-g)(x) =$ ______________________ Domain: ______________

$(fg)(x) =$ ______________________ Domain: ______________

$\left(\dfrac{f}{g}\right)(x) =$ ______________________ Domain: ______________

(EX) Given $f(x) = \dfrac{1}{x^2-4}$, $g(x) = \sqrt{x}$, and $h(x) = 5x-2$, find the following:

(a) Domain of f

(b) Domain of g

(c) Domain of h

(d) $(f+g)(4)$

(e) $(gh)(9)$

(f) $(h/f)(0)$

(g) $(f/g)(-1)$

(h) $(gg)(x)$ and its domain

(i) $(f/h)(x)$ and its domain

(j) $(g-h)(x)$ and its domain

WeBWorK

Suppose that $f(x) = \sqrt{x+3}$ and $g(x) = 2x^2 - 128$. First find the following:

(a) $(f+g)(3) =$

Now find the domain of the following two functions

(b) $(f+g)(x)$

(c) $(f/g)(x)$

and write your answer using interval notation.

(b) Domain:

(c) Domain:

Difference Quotient: The quantity below is very important in Calculus, but we will use it as a way to practice function notation and function arithmetic. In working these problems, we want to simplify them until we get to the point where the 'h' in the definition of the difference quotient cancels from the denominator (although other instances of 'h' may remain).

The **difference quotient** of a function f is: $\dfrac{f(x+h)-f(x)}{h}$

(EX) Find and simplify the difference quotient for the following functions:

(a) $f(x) = 8x - 7$

(b) $f(x) = \dfrac{1}{x+2}$

(c) $f(x) = 2 - 5x - x^2$

WeBWorK

Find the difference quotient of the function

$$f(x) = \frac{7}{x}$$

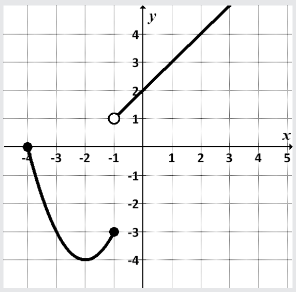
and simplify your answer as much as possible.

$$\frac{f(x+h) - f(x)}{h} = \frac{\square}{\square}$$

1.6: Graphs of Functions

Graphing Functions: If $f(a) = b$, then the ordered pair ________________ is on the graph of $y = f(x)$.

(EX) Given the graph below, find:

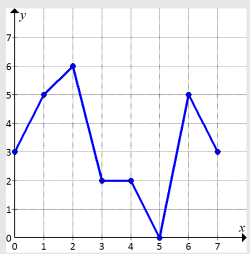


(a) $f(-3) =$ ________ (b) $f(-1) =$ ________

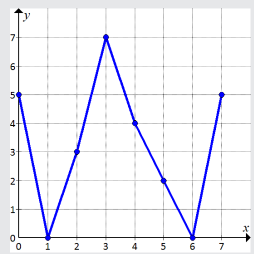
(c) $f(2) =$ ________ (d) $f($_____$) = 0$

(e) $f($_____$) = 2$ (f) $f(1) - f(-2) =$ ________

(EX) Given the graphs of $y = f(x)$ and $y = g(x)$ below, find the function values.



$y = f(x)$



$y = g(x)$

(a) $(f + g)(0)$ (b) $(f - g)(2)$ (c) $(fg)(4)$

(d) $(\frac{f}{g})(5)$ (e) $(g - f)(6)$ (f) $(\frac{g}{f})(3)$

Finding Intercepts: It is often useful to find points where the graph of a function crosses the x-axis or the y-axis. The ability to find these points is an important skill that will be used throughout the semester.

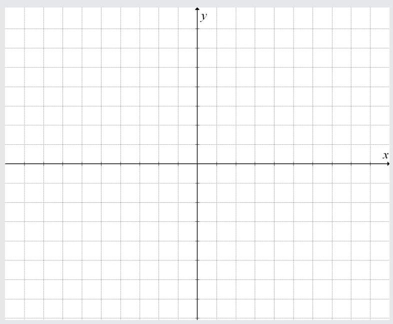
Finding the Intercepts of the Graph of a Function

x-intercepts	**y-intercepts**
A point of the form ________________	A point of the form ________________
To find the x-intercept, solve ________.	To find the y-intercept, find ________.

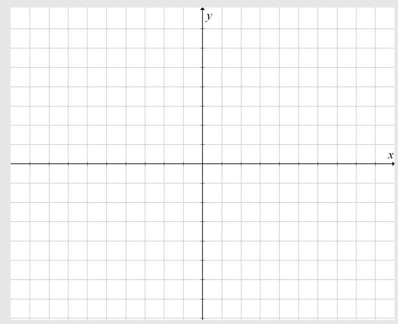
Zeros: The zeros of a function f are the solutions to the equation ________________. If the zero is a real number, then it corresponds to an __________________________ on the graph of $y = f(x)$.

(EX) Sketch the graph of each function given below. State the domain of the function, and identify the intercepts. What are the zeros of the function?

(a) $f(x) = \dfrac{x+1}{4}$



(b) $g(x) = x^2 - 4$



WeBWorK

Find the domain, x-intercept(s), y-intercept(s), and zero(s) of the function

$$f(x) = \sqrt{7 - x}.$$

Write the domain in interval notation, and write **none** in the corresponding blank if no such intercept or zero exists.

(a) Domain: ________

(b) x-intercept(s): ________

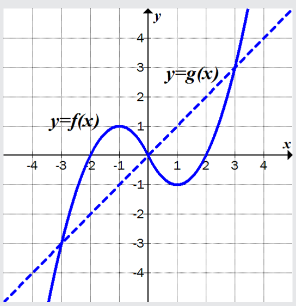
(c) y-intercept(s): ________

(d) Zero(s): ________

Solving Equations and Inequalities Graphically: Suppose f and g are functions whose graphs are given.

Equation/Inequality to Solve:	Solution Set:
$f(x) = 0$	x-values where the graph of $f(x)$ crosses the x-axis.
$f(x) > 0$	x-values where the graph of $f(x)$ is above the x-axis.
$f(x) < 0$	x-values where the graph of $f(x)$ is below the x-axis.
$f(x) = g(x)$	x-values where the graph of $f(x)$ and $g(x)$ intersect.
$f(x) > g(x)$	x-values where the graph of $f(x)$ is above $g(x)$.
$f(x) < g(x)$	x-values where the graph of $f(x)$ is below $g(x)$.

(EX) Given the graphs of two functions $y = f(x)$ (solid), and $y = g(x)$ (dashed) shown below, solve each equation/inequality. (Use interval notation when appropriate.)



(a) $f(x) = 0$

(b) $f(x) \geq 0$

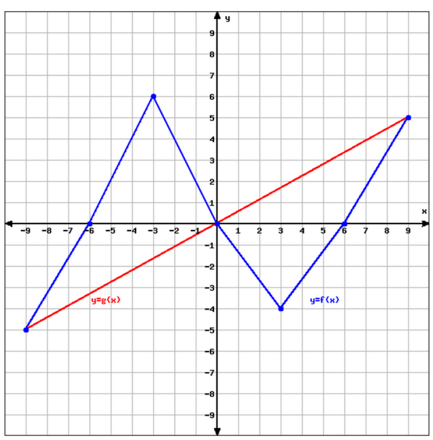
(c) $f(x) < 0$

(d) $f(x) = g(x)$

(e) $f(x) \leq g(x)$

(f) $f(x) > g(x)$

WeBWorK



Given the graph of $y = f(x)$ (drawn in blue) and $y = g(x)$ (drawn in red) above, solve each of the following equations/inequalities for x.

(a) Solve $f(x) = g(x)$:

(b) Solve $f(x) > g(x)$:

(c) Solve $f(x) \leq g(x)$:

(a) Solve $g(x) = 0$:

(b) Solve $g(x) \geq 0$:

(c) Solve $f(x) < 0$:

Symmetry: The function f is symmetric:

- about the y-axis if and only if ______________________________for all x in the domain of f.
- about the origin if and only if ______________________________for all x in the domain of f.

Why don't we mention symmetry of a function about the x-axis?

When discussing the symmetry of functions, we often use other terminology:

EVEN function	**ODD function**
Symmetric about the ________________	Symmetric about the ________________
Has the property that $f(-x) =$________	Has the property that $f(-x) =$________

How to Determine if a Function is Even, Odd, or Neither:

1. Find $f(-x)$, and simplify this as much as possible.
2. Ask yourself:
   (a) Does $f(-x) = f(x)$?
   (Does the answer from Step 1 equal the original function?) If so, then $f(x)$ is EVEN.
   (b) Does $f(-x) = -f(x)$?
   (Does the answer from Step 1 equal the opposite of the original function?) If so, then $f(x)$ is ODD.
   (c) If neither of the above is true, then $f(x)$ is NEITHER even nor odd.

(EX) Determine analytically whether the function is even, odd, or neither.

(a) $f(x) = 5x^4 + 2x^2$

(b) $g(x) = x^3 + 1$

(c) $h(x) = \dfrac{x^2+1}{x}$

WeBWorK

Let $f(x) = \dfrac{x}{x^5 - x^3 + 10}$.

Determine $f(-x)$ first and then determine whether the function is even, odd, or neither. Write **even** if the function is even, **odd** if the function is odd, and **neither** if the function is neither even nor odd.

$f(-x) =$ __________

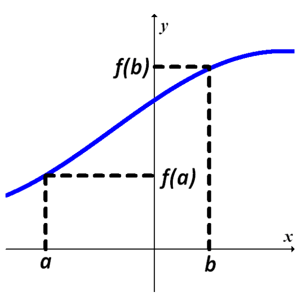
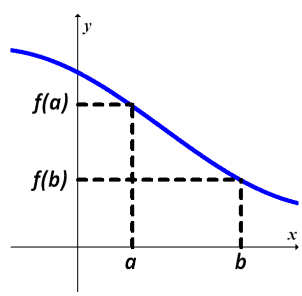
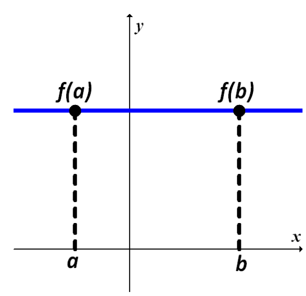
Even/Odd/Neither: __________

Increasing, Decreasing, and Constant Functions:

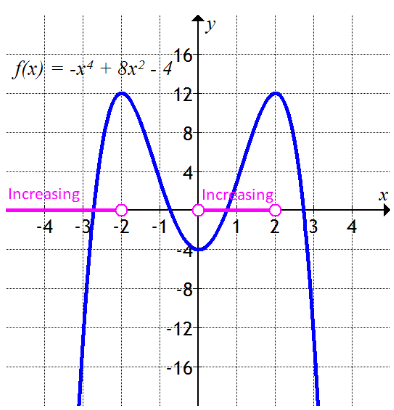
A function f is said to be:

INCREASING	DECREASING	CONSTANT

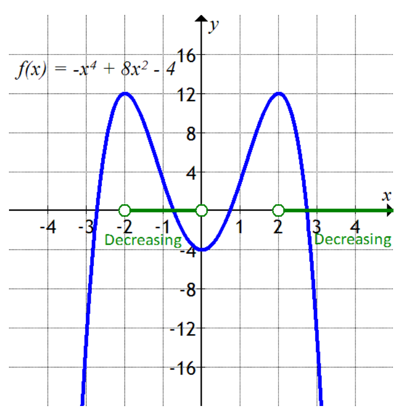
on an open interval I if for all $a, b \in I$, $a < b$ implies that:

__________	__________	__________
		

For example, consider the function $f(x) = -x^4 + 8x^2 - 4$ shown below.

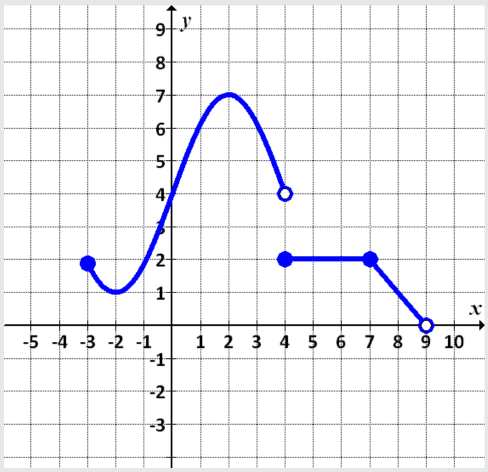


Increasing on $(-\infty, -2), (0, 2)$



Decreasing on $(-2, 0), (2, \infty)$

(EX) Given the graph of $f(x)$ below, find the following:



(a) Domain

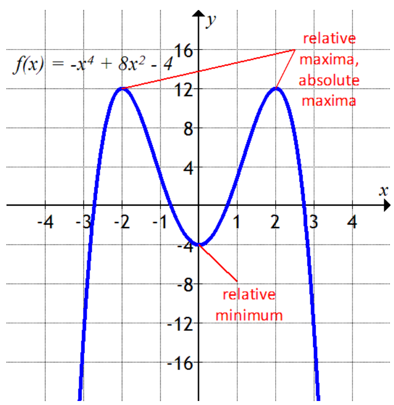
(b) Range

(c) Interval(s) on which f is increasing

(d) Interval(s) on which f is decreasing

(e) Interval(s) on which f is constant

Relative/Local Maxima and Minima:



The y-value $f(c)$ is a ____________________________of the function f if it is the highest point on the graph in some open interval surrounding c.

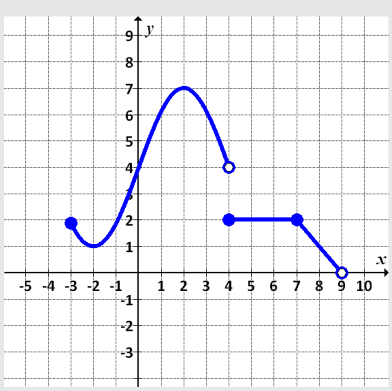
Similarly, the y-value $f(c)$ is a ____________________________of the function f if it is the lowest point on the graph in some open interval surrounding c.

Absolute Maximum and Minimum:

The y-value $f(c)$ is the ____________________________of the function f if it is the highest point on the graph for all x in the domain of f.

Similarly, the y-value $f(c)$ is the ____________________________of the function f if it is the lowest point on the graph for all x in the domain of f. (Sometimes these are simply referred to as the maximum and minimum.)

(EX) Given the graph of $f(x)$ below, find the following:



(a) Relative maximum of _____, which occurs at $x =$ _____

(b) Absolute maximum of _____, which occurs at $x =$ _____

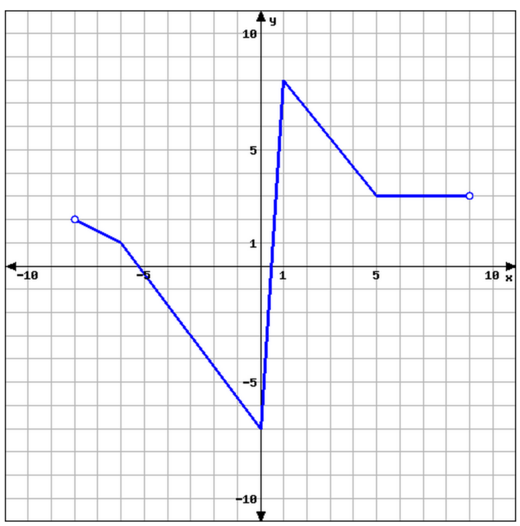
(c) Relative minimum of _____, which occurs at $x =$ _____

(d) Absolute minimum of _____, which occurs at $x =$ _____

Summary of Graph Information:

To Find:	Solve/Determine:	Answer Format:
x-intercepts	Solve $f(x) = 0$	ordered pair $(x, 0)$
y-intercepts	Find $y = f(0)$	ordered pair $(0, y)$
zeros	Solve $f(x) = 0$	List of x-values
Solution to $f(x) = g(x)$	Where f and g intersect	List of x-values
Solution to $f(x) > 0$	Where f is above x-axis	Interval of x-values
Solution to $f(x) < 0$	Where f is below x-axis	Interval of x-values
Domain	x-values contained in the graph	Interval of x-values
Range	y-values contained in the graph	Interval of y-values
Increasing	Where graph goes up from L to R	OPEN Interval of x-values
Decreasing	Where graph goes down from L to R	OPEN Interval of x-values
Constant	Where graph stays flat from L to R	OPEN Interval of x-values
Relative Max	Highest y-value on some open interval	List of y-values
Relative Min	Lowest y-value on some open interval	List of y-values
Absolute Max	Highest y-value on the entire graph	Single y-value
Absolute Min	Lowest y-value on the entire graph	Single y-value

WeBWorK



Given the graph of $f(x)$ above, find the following and write your answers using interval notation:

(a) Domain:

(b) Range:

(c) Interval(s) on which $f(x)$ is increasing:

(d) Interval(s) on which $f(x)$ is decreasing:

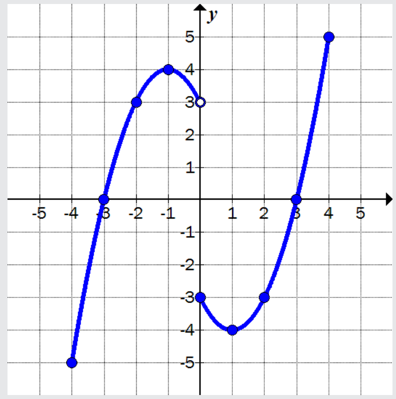
(e) Interval(s) on which $f(x)$ is constant:

(f) Local maxima:

(g) Local minima:

Help: Write "none" if no such interval(s) exist or if there is no relative maximum or relative minima. Separate multiple intervals by a comma when entering intervals for increasing, decreasing, and constant. Click here for help entering intervals.

(EX) Use the graph of $y = f(x)$ shown below to answer the questions.



(a) Find the domain of f.

(b) Find the range of f.

(c) Determine $f(-4)$.

(d) Solve $f(x) = 5$.

(e) List the x-intercepts, if any.

(f) List the y-intercepts, if any.

(g) Find the zeros of f.

(h) Solve $f(x) \geq 0$.

(i) Find the number of solutions to $f(x) = 3$.

(j) Does f appear to be even, odd, or neither?

(k) List the intervals where f is increasing.

(l) List the intervals where f is decreasing.

(m) Find the local maximums, if any.

(n) Find the local minimums, if any.

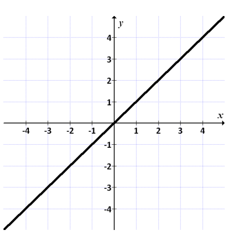
(o) Find the maximum, if any.

(p) Find the minimum, if any.

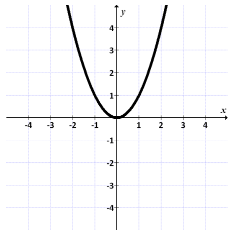
1.7: Transformations

In this section, we will learn how to transform the graph of a known function through a combination of shifts, reflections, and scaling.

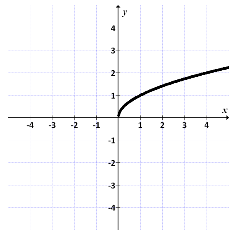
Some Basic Graphs You Need To Know



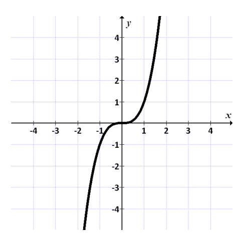
$y =$ __________



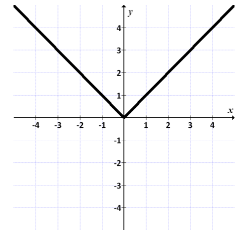
$y =$ __________



$y =$ __________



$y =$ __________



$y =$ __________

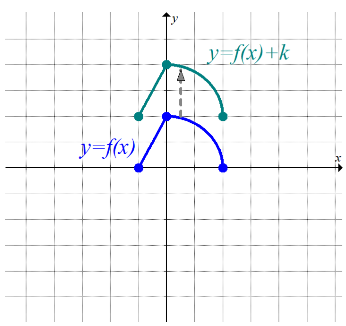
TRANSFORMATIONS: Shifts/Translations

Let $h > 0$ and $k > 0$. Then, the graph of $f(x)$ can be shifted to produce the graphs described below.

$\boxed{y = f(x) + k}$

VERTICAL SHIFT **UP** k UNITS:

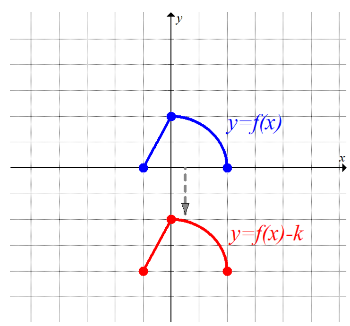
Changes a point on the graph from (x, y) to __________.



$\boxed{y = f(x) - k}$

VERTICAL SHIFT **DOWN** k UNITS:

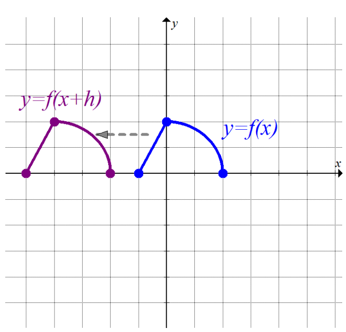
Changes a point on the graph from (x, y) to __________.



$\boxed{y = f(x + h)}$

HORIZONTAL SHIFT **LEFT** h UNITS

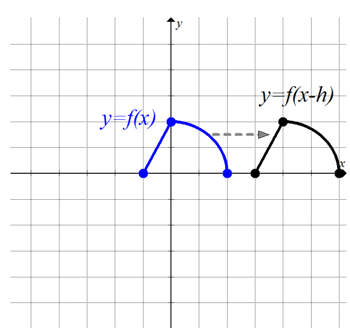
Changes a point on the graph from (x, y) to __________.



$\boxed{y = f(x - h)}$

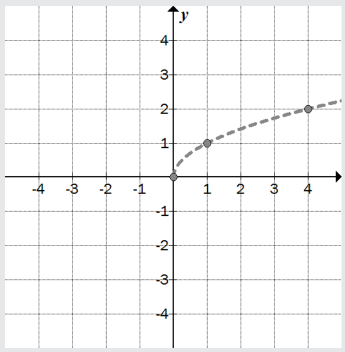
HORIZONTAL SHIFT **RIGHT** h UNITS:

Changes a point on the graph from (x, y) to __________.

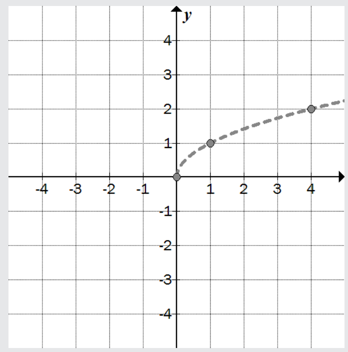


(EX) Use the graph of $y = \sqrt{x}$ to sketch the graph of each transformed function listed below:

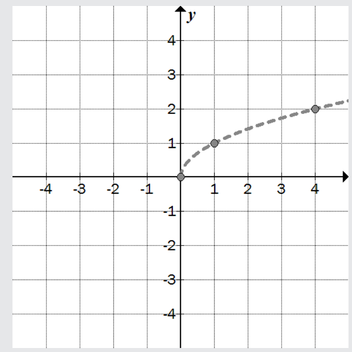
(a) $y = \sqrt{x} + 2$



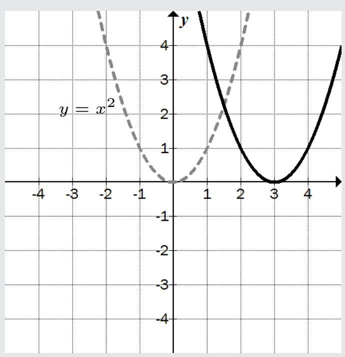
(b) $y = \sqrt{x - 2}$



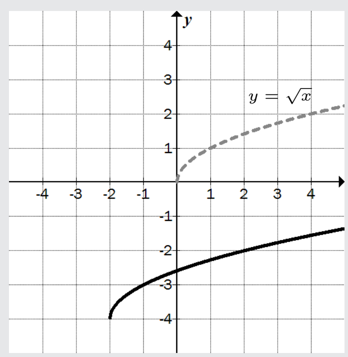
(c) $y = \sqrt{x+1} - 3$



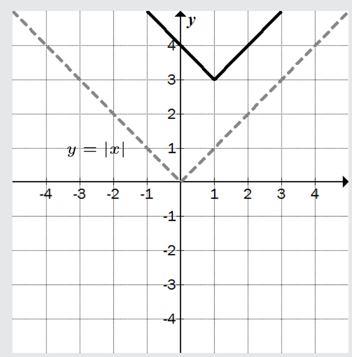
(EX) The graph below represents a translation of the graph of $y = x^2$, $y = \sqrt{x}$, or $y = |x|$. Find the equation of the function which is graphed.



$y =$____________



$y =$____________



$y =$____________

WeBWorK

Suppose $(-2, 5)$ is on the graph of $y = f(x)$. Find the corresponding point on the graph of the given transformed function.

(a) $y = f(x - 3)$:

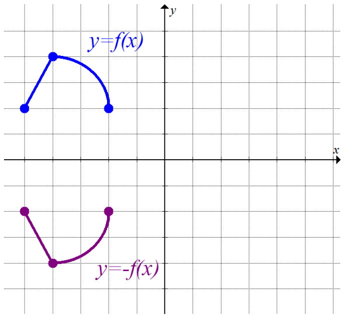
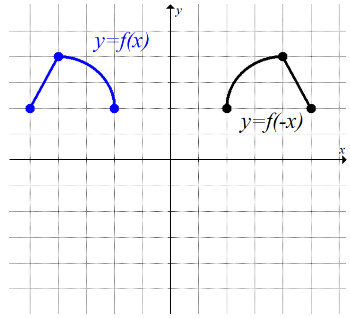
(b) $y = f(x + 3)$:

(c) $y = f(x) - 3$:

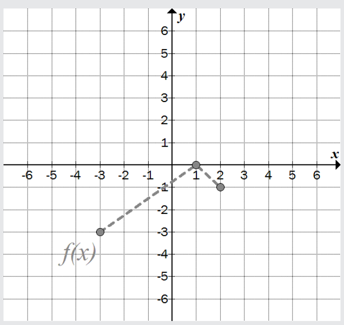
(d) $y = f(x) + 3$:

(e) $y = f(x + 5) + 6$:

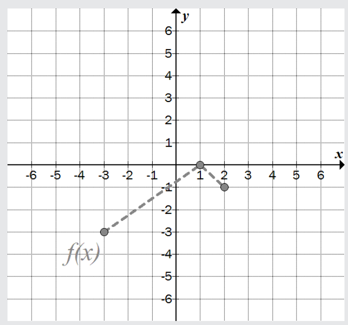
TRANSFORMATIONS: Reflections

$\boxed{y = -f(x)}$	$\boxed{y = f(-x)}$
VERTICAL REFLECTION OVER THE **X-AXIS**:	HORIZONTAL REFLECTION OVER THE **Y-AXIS**:
Changes a point on the graph from (x, y) to __________.	Changes a point on the graph from (x, y) to __________.
	

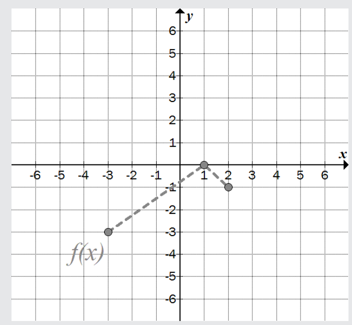
(EX) Use the graph of $f(x)$ given below to sketch a graph of each equation.



$y = -f(x+2)$



$y = f(-x) - 3$



$y = -f(-x) + 2$

WeBWorK

Suppose $(-6, -6)$ is on the graph of $y = f(x)$. Find the corresponding point on the graph of the given transformed function.

(a) $y = f(-x)$: __________

(b) $y = -f(x)$: __________

(c) $y = -f(x+4) + 1$: __________

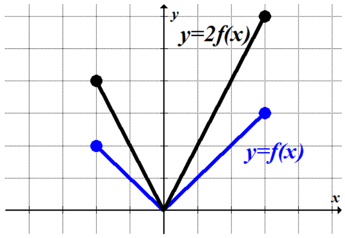
(d) $y = f(-x) - 5$: __________

TRANSFORMATIONS: Stretching and Shrinking

$y = af(x)$, where $|a| > 1$

VERTICAL **STRETCH** BY A FACTOR OF **a**:

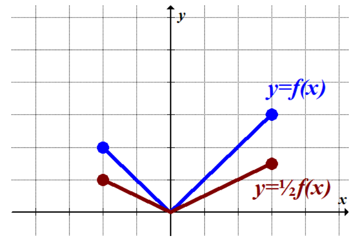
Changes a point from (x, y) to ________.



$y = af(x)$, where $0 < |a| < 1$

VERTICAL **SHRINK** BY A FACTOR OF **a**:

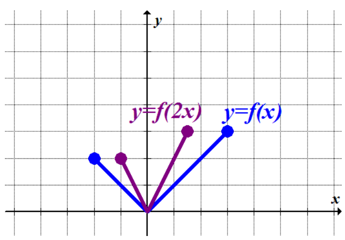
Changes a point from (x, y) to ________.



$y = f(bx)$, where $|b| > 1$

HORIZONTAL **SHRINK** BY A FACTOR OF **1/b**:

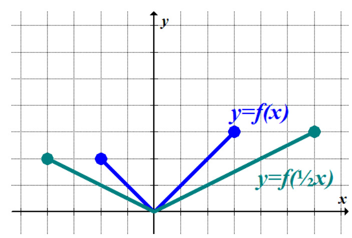
Changes a point from (x, y) to ________.



$y = f(bx)$, where $0 < |b| < 1$

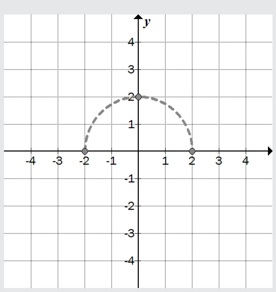
HORIZONTAL **STRETCH** BY A FACTOR OF **1/b**:

Changes a point from (x, y) to ________.

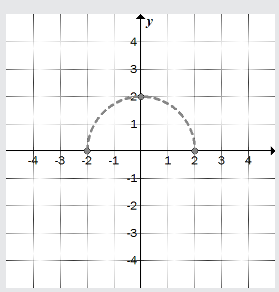


(EX) Given the graph of $f(x)$ (a semicircle), sketch the graph of the transformed function below:

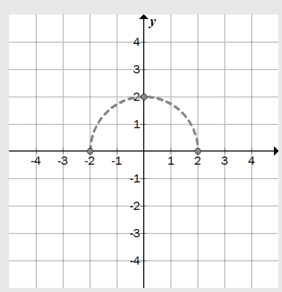
(a) $y = 2f(x)$



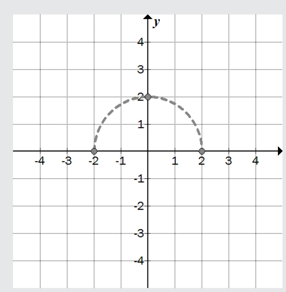
(b) $y = \frac{1}{2}f(x)$



(c) $y = f(2x)$



(d) $y = f(\frac{1}{2}x)$



WeBWorK

Suppose $(6, -3)$ is on the graph of $y = f(x)$. Find the corresponding point on the graph of the given transformed function.

(a) $y = f(3x)$: ________

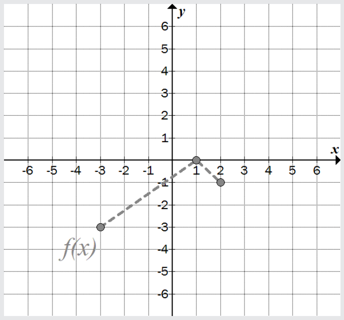
(b) $y = 3f(x)$: ________

(c) $y = f(x/3)$: ________

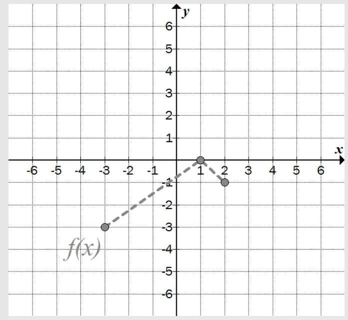
(d) $y = \frac{1}{3}f(x)$: ________

(e) $y = 5f(6x)$: ________

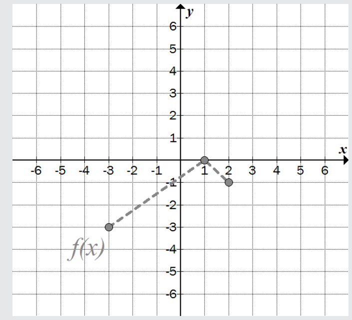
(EX) Use the graph of $f(x)$ given to sketch a graph of each equation.



$y = f(2x) - 1$



$y = -2f(x+1)$

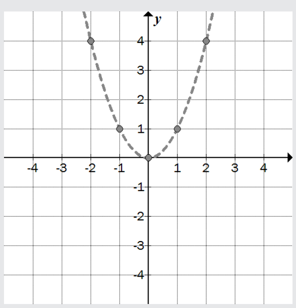


$y = f(-\frac{1}{2}x)$

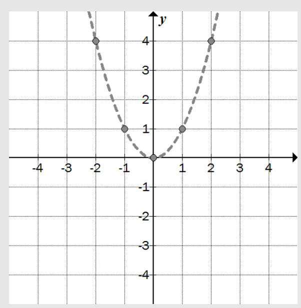
TRANSFORMATIONS: Order of Operations

(EX) Does the order in which you apply transformations matter? Sketch the transformations of $y = x^2$ below, using the order given. Do the graphs end up being the same?

(1) Reflect over the x-axis.
(2) Shift up 2 units.



(1) Shift up 2 units.
(2) Reflect over the x-axis.



A more detailed discussion of order of operations for transformations is available in the online textbook. For this course, we are primarily interested in the order of operations for vertical transformations. When you encounter something like $-3f(x)+4$, it is important that you perform the reflection and stretch before you shift the graph up. Reversing the order (doing the shift first) would result in an incorrect graph.

(EX) Suppose that the point $(2, -7)$ is on the graph of $f(x)$. Find a point on the graph of:

(a) $y = f(-3x) + 4$

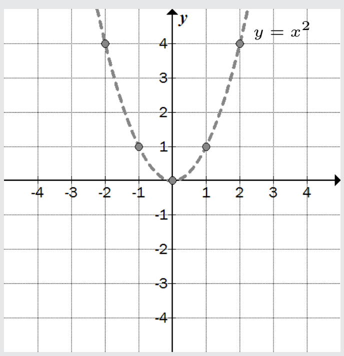
(b) $y = -2f(x-5) - 1$

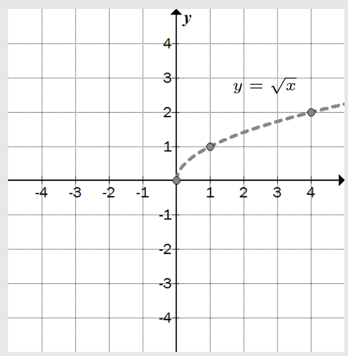
(EX) Write a formula for a function g whose graph is similar to $f(x) = 2\sqrt{x} + x$, but satisfies the given conditions.

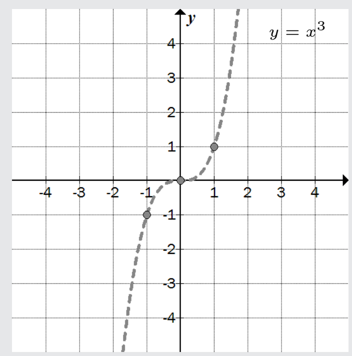
(a) Reflected about the y-axis, shrunk vertically by a factor of $\frac{1}{4}$, and moved up 2 units.

(b) Reflected about the x-axis, shifted right 10 units, and down 3 units.

(EX) Use transformations to sketch a graph of each function.







$y = 1 - (x-3)^2$

$y = 3\sqrt{x+1} - 3$

$y = -\frac{1}{3}(x-2)^3 + 2$

WeBWorK

Suppose $(4, 3)$ is on the graph of $y = f(x)$. Find the corresponding point on the graph of the given transformed function.

(a) $y = -7f(-x/4)$:

(b) $y = \frac{5}{4}f(x+7) + 4$:

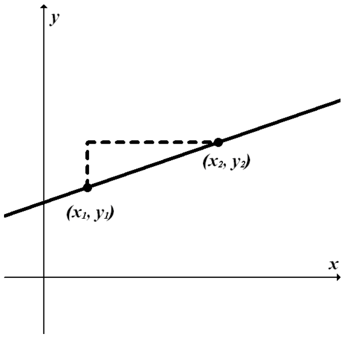
(c) $y = -\frac{1}{4}f(-5x) + 4$:

TRANSFORMATIONS: A Summary

	Horizontal or Vertical?	Shift/Reflection/ Scaling?	Description	Changes the point (x, y) to:
$f(x) + k$				
$f(x) - k$				
$f(x + h)$				
$f(x - h)$				
$-f(x)$				
$f(-x)$				
$af(x), \|a\| > 1$				
$af(x), 0 < \|a\| < 1$				
$f(bx), \|b\| > 1$				
$f(bx), 0 < \|b\| < 1$				

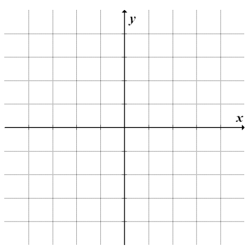
2.1: Linear Functions

Slope: The slope m of a line containing the points (x_1, y_1) and (x_2, y_2) is given by:

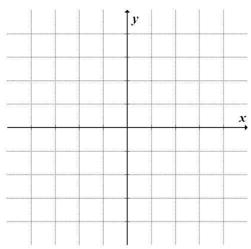


Slope can be thought of as the ____________________________of y with respect to x.

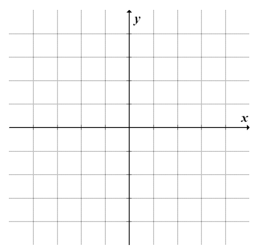
Positive Slope



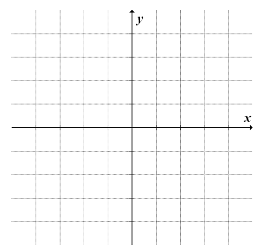
Negative Slope



Zero Slope



Undefined Slope



(EX) Find the slope of the line:

(a) containing $(-6, -\frac{1}{2})$ and $(-\frac{13}{2}, 2)$

(b) $f(-4) = -5$ and $f(4) = 3$

(c) containing (p, q) and $(p, -q)$

Parallel and Perpendicular Lines:

Two lines are ____________________________if they have the same slope.
(Note that vertical lines are parallel to each other even though their slopes are undefined.)

Two lines with non-zero slopes m_1 and m_2 are ____________________________if $m_1 \cdot m_2 = -1$.
(Note that vertical lines are perpendicular to horizontal lines and vice versa.)

(EX) Find the value of k so that the line containing the points $(1, 3)$ and $(7, 4)$ is parallel to the line containing the points $(-4, -5)$ and $(k, 2)$.

Repeat the problem, for perpendicular lines.

WeBWorK

Find the slope of the line containing the points $\left(-\frac{2}{5}, -\frac{3}{4}\right)$ and $\left(-\frac{1}{4}, \frac{5}{2}\right)$ if it exists or write **undefined** if the slope is undefined. You must reduce your answer completely.

$m =$ ______

Find the value of k so that the line containing the points $(7, -7)$ and $(-6, k)$ is perpendicular to the line containing the points $(-5, 5)$ and $(-8, 0)$.

$k =$ ______

THE EQUATION OF A LINE:

Horizontal and Vertical Lines

- The graph of the equation ________________ is a ________________ line through the point $(a, 0)$.
- The graph of the equation ________________ is a ________________ line through the point $(0, b)$.

Point-Slope Form of a Line: We can rewrite our formula for slope to derive an equation of a line passing through a point (x_0, y_0), with slope m:

Slope-Intercept Form of a Line: The equation of a line with slope m and y-intercept $(0, b)$ is given by

__________________________. If a linear equation is given to us in another form, we can rewrite it in this form by solving the equation for y.

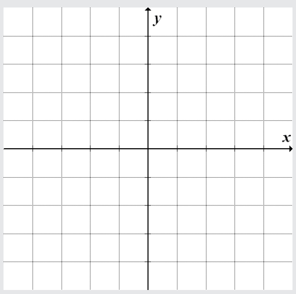
LINEAR FUNCTIONS:

A linear function is a function of the form ______________________________, where $m, b \in \mathbb{R}$. The domain of a linear function is ______________________________. If $m = 0$, then the linear function has the form $f(x) = b$, which is called the ______________________________. The graph of a constant function is a horizontal line.

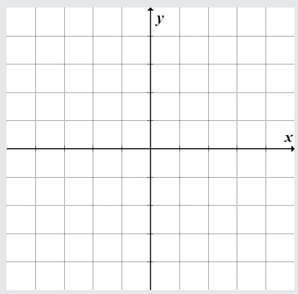
Does every line represent a function?

(EX) Find the slope and y-intercept, and graph the line:

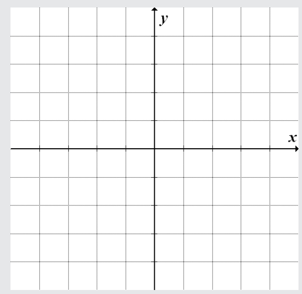
(a) $f(x) = 3 - \frac{2}{3}x$



(b) $2x - 3y = 12$



(c) $f(x) = \frac{3}{5}$



(EX) Find the slope-intercept form of the equation of the line with the following properties:

(a) $m = -\frac{3}{8}$, y-intercept $(0, -7)$

(b) $m = \frac{2}{7}$, passes through $(-2, 5)$

(c) passes through $(-1, -3)$ and $(4, -9)$

(d) Horizontal, passing through $(-2, 11)$

(e) $8 = f(2)$, $-3 = f(3)$

(f) perpendicular to $y = 3$, passing through $(-1, 1)$

WeBWorK Find the equation of the line through the points $\left(\frac{4}{7}, -2\right)$ and $(4, -1)$. Write your answer in the form $y = mx + b$.

(EX) Are the lines parallel, perpendicular, or neither?

(a) $y = 5x - 1$, $y = \frac{1}{5}x + 1$

(b) $y = x + 7$, $x + y = 1$

(c) $y = 2x - 5$, $2x - y = 7$

(EX) Given the point $P(5, -1)$, and the lines $L_1 :\ y = \frac{3}{4}x - 1$ and $L_2 :\ 7x - 2y = 15$, find the equation of the line:

(a) passing through P, and perpendicular to L_1

(b) passing through P, and parallel to L_2

(c) passing through P and parallel to the y-axis

WeBWorK

Find the equation of the line though the point $(-3,-2)$ and perpendicular to the line $y=-\frac{4}{5}x+8$. Write your answer in the form $y=mx+b$

Applications:

(EX) Your cable company charges a $65 installation fee and $80 per month for service.

(a) Write a function for $C(t)$, the cost of t months of cable service.

(b) What is the total cost for 8 months of service?

(c) What are the domain and range of this function?

(EX) An on-demand publisher charges $25.50 to print a 600 page book and $17.50 to print a 400 page book. Find a linear function which models the cost of a book C as a function of the number of pages p. Interpret the slope of the linear function and find and interpret $C(0)$.

WeBWorK

Alphonse is a salesman that earns a base salary of $\$1{,}400.00$ per month and a commission of 5% on the amount of sales he makes. One month Alphonse received a paycheck for $\$1{,}780.00$ Find the amount of his sales for the month.

Amount of Sales in Dollars: (Round your answer to the nearest cent and include units.)

2.3: Quadratic Functions

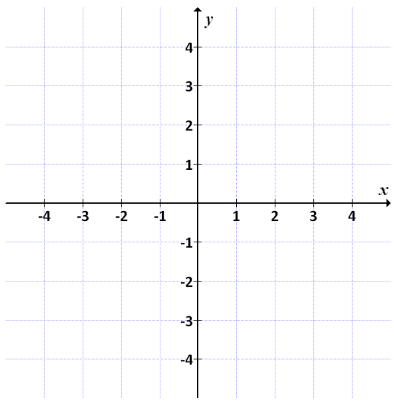
Quadratic Functions:

A quadratic function is a function of the form __________________________, where $a, b, c \in \mathbb{R}, a \neq 0$.

The domain of a quadratic function is ________________.

The graph of a quadratic function is called a ________________.

The most basic quadratic function is $f(x) = x^2$.



Domain: ________________

Range: ________________

Increasing on: ________________

Decreasing on: ________________

Max/Min: ________________

x-intercept(s): ________________

y-intercept: ________________

Graphs of Quadratic Functions: In this section, we want to examine how to graph quadratic functions and identify the characteristics of the graph. First, let's recall what we know about finding zeros and intercepts:

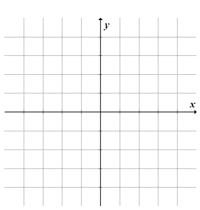
Finding zero(s)/x-intercept(s): Solve $f(x) = 0$ Finding y-intercept: Evaluate $f(0)$

(EX) Find the zero(s), x-intercept(s) and y-intercept of the quadratic function $g(x) = 5x^2 - 10x + 1$.

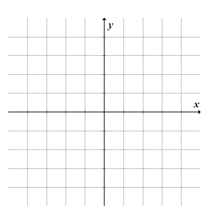
Does every quadratic function have two zeros? Does every parabola have two x-intercepts?

Recall: The Discriminant is defined as $D =$________________. This value helps us to 'discriminate' between the different outcomes that we can have for a quadratic function and its zeros.

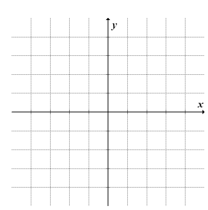
If $D > 0$



If $D = 0$



If $D < 0$

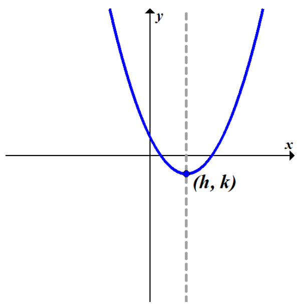


In addition to the intercepts, another point on the graph of a quadratic function is useful to us when graphing the parabola:

Vertex: The vertex of a parabola is the point (h, k) on the graph where the function changes from increasing to decreasing, or vice versa. The vertex will also be the absolute maximum or the absolute minimum of the quadratic function.

Once we know the vertex, we can use that information to help determine basic information about the graph of the quadratic function:

$\boxed{a > 0}$



Domain: ________________

Range: ________________

Vertex: ________________

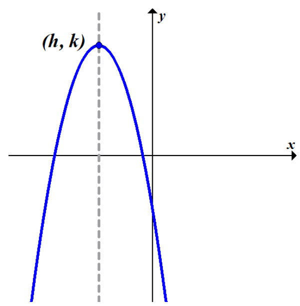
Increasing on: ________________

Decreasing on: ________________

Max/Min?: ________________

Axis of Symmetry: ________________

$\boxed{a < 0}$



Domain: ________________

Range: ________________

Vertex: ________________

Increasing on: ________________

Decreasing on: ________________

Max/Min?: ________________

Axis of Symmetry: ________________

We generally work with quadratic functions in one of two forms:

Vertex Form	**Standard Form**
$f(x) =$ ________________________,	$f(x) =$ ________________________,
where $a, h, k \in \mathbb{R}$, $a \neq 0$	where $a, b, c \in \mathbb{R}$, $a \neq 0$
Vertex: ________________	Vertex: ________________

(EX) Find the vertex of the quadratic function.

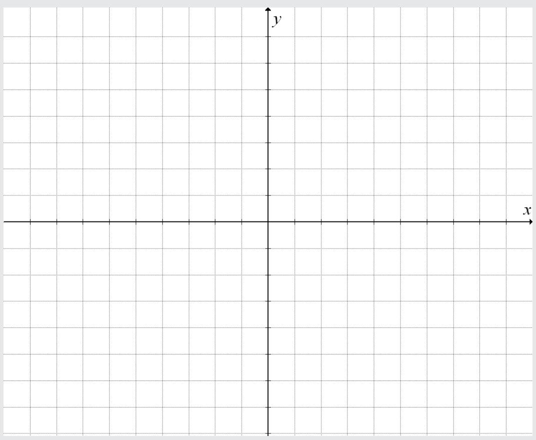
(a) $f(x) = -4(x-7)^2 - 5$

(b) $g(x) = 10x^2 + 40x + 37$

Now, we can combine all of this information to help us construct the graph of a quadratic function and to identify basic information about the graph.

(EX) Sketch the graph of the function, and find the other information listed below.

(a) $f(x) = -(x+3)^2 - 4$



Domain: ________________

Range: ________________

Vertex: ________________

Increasing on: ________________

Decreasing on: ________________

Max/Min?: ________________

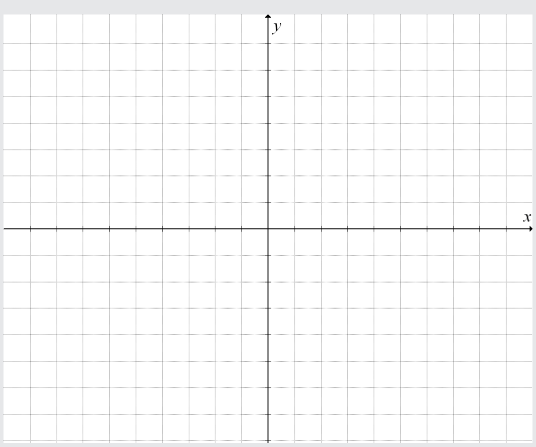
Axis of Symmetry: ________________

Zero(s): ________________

x-intercept(s): ________________

y-intercept: ________________

(b) $g(x) = 3x^2 + 12x + 8$



Domain: ________________

Range: ________________

Vertex: ________________

Increasing on: ________________

Decreasing on: ________________

Max/Min?: ________________

Axis of Symmetry: ________________

Zero(s): ________________

x-intercept(s): ________________

y-intercept: ________________

WeBWorK

Find the following information about the function $f(x) = -\frac{x^2}{2} + 4x - 6$.

(a) The vertex is the point .

(b) The axis of symmetry is the line .

(c) The function is increasing on the interval .

(d) The function is decreasing on the interval .

(e) The function takes its select value at $y =$.

(f) The domain of the function is the interval .

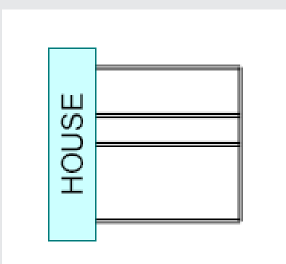
(g) The range of the function is the interval .

(h) The x-intercepts of f, if any exist, are .

(i) The y-intercept of f, if it exists, is .

Applications of Quadratic Functions:

(EX) Max wishes to enclose a rectangular garden, using one side of his house as shown below. What is the maximum area that he can enclose if he has 80 ft of fence to use?



(EX) The sum of the base and the height of a triangle is 22 cm. Find the dimensions for which the area is a maximum.

WeBWorK

Suppose $C(x) = x^2 - 8x + 24$ represents the costs, in *hundreds*, to produce x *thousand* pens. How many pens should be produced to minimize the cost? What is the minimum cost?

Number of pens to minimize cost: ______ pens

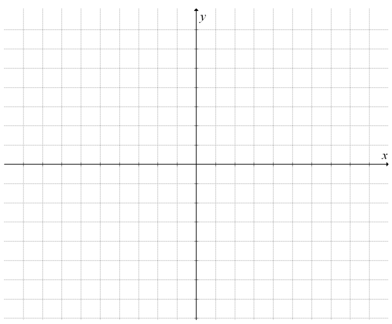
Minimum cost: ______ dollars

2.4: Inequalities with Quadratic Functions

Quadratic Inequalities:

When solving quadratic inequalities, it is easiest for us to always compare with 0. In this way, we limit our analysis to simply determining when the function takes on values that are positive (above the x-axis), negative (below the x-axis), or zero (crossing the x-axis). We can do this by using either a graph, or a sign diagram (also called a sign chart).

To Solve a Quadratic Inequality Graphically: Solve $x^2 - 4x < 12$.



To Solve a Quadratic Inequality Using a Sign Diagram: Solve $x^2 - 4x < 12$.

STEP 1. Write the inequality with a quadratic function $f(x)$ on one side, and 0 on the other side.

STEP 2. Find the zeros of f, and put them on a number line.

STEP 3. Choose a test point in each interval from your number line.

STEP 4. Determine whether f is positive or negative when evaluated at each test point.

STEP 5. Choose the intervals which correspond to the correct sign to solve the inequality.

(EX) Solve. Write your answer in interval notation.

(a) $x^2 < 25$

(b) $3x^2 \leq 2 - x$

(c) $x^2 - 6x > -9$

(d) $x^2 + 4 \geq 1$

WeBWorK

Solve the following inequality and write your answer using interval notation.

$$y^2 + 7y + 11 \geq -3y - 5$$

Answer:

(EX) Find the domain of the function: $f(x) = \sqrt{3x^2 - 11x - 4}$

(EX) The temperature F, in degrees Fahrenheit, t hours after 6 AM is given by $F(t) = -\frac{1}{2}t^2 + 8t + 32$, for $0 \leq t \leq 12$. When is it warmer than 46° F?

WeBWorK

Suppose $C(x) = 10x^2 - 50x + 61$, $x \geq 0$ represents the cost, in hundreds of dollars, to produce x thousands of pens. Find the number of pens which can be produced for no more than $\$2100$.

Answer: between ______ thousand and ______ thousand pens

3.1: Polynomial Functions

Polynomial Functions: A polynomial function has the form:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where the coefficients $a_0, \ldots a_n$ are real numbers.

- Leading term: ________________
- Leading coefficient: ________
- Degree: ________
- Constant Term: ________

We can classify a function by its degree:

Degree	Classification	Example
0	________________	__________________________
1	________________	__________________________
2	________________	__________________________
3	________________	__________________________
4	________________	__________________________

End Behavior - The Leading-Term Test: If a polynomial has leading term $a_n x^n$, then its end behavior can be modeled by:

	Degree (n) even	Degree (n) odd
Leading Coefficient positive ($a_n > 0$)		
Leading Coefficient negative ($a_n < 0$)		

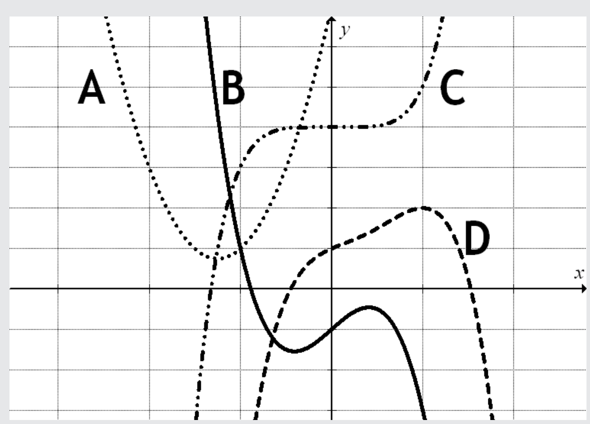
(EX) Use the leading-term test to match the graph to the function shown below:

(i) $f(x) = -4x^3 + 2x - 1$

(ii) $f(x) = 1 + x^5$

(iii) $f(x) = -2x^4 + 3x^3 - x^2 + x + 1$

(iv) $f(x) = 4x^2 + 10x + 7$



If a polynomial is given to us in factored form, then we can find the leading term by multiplying together the leading term of each factor.

(EX) Use the leading-term test to sketch the end behavior of the function.

(a) $P(x) = -2(x+5)^3(x-4)(x+1)^2$ (b) $P(x) = (x+4)^2(1-3x)(x^2+1)^3$

WeBWorK

Determine the end behavior of the following polynomial function:

$$f(x) = -\frac{8}{9}x^{10}(x-9)^2(x-3)$$

The leading term of the polynomial is ______

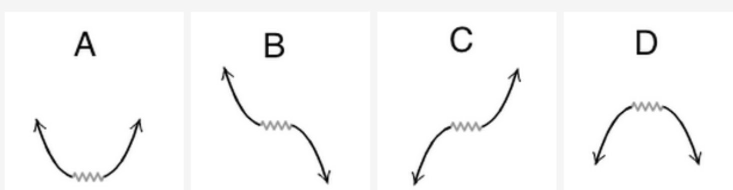
The degree of $f(x)$ is ______

The leading coefficient is ______

The end behavior of the polynomial $f(x)$ is of the form:

- A
- B
- C
- D

where:



Zeros:

RECALL: To find the zeros of a function, solve the equation ______________.

(eg) $f(x) = x^2 + 4x + 3$

Factors: ______________________

Zeros: ______________________

x**-intercepts:** ______________________

Multiplicity:

If a polynomial function has a factor of ________________, then it has a corresponding zero of $x =$________.

If a factor is repeated, we say that its corresponding zero has __________________________.

(eg) $f(x) = (x-1)^4(x+2)^3(x-7)$

Zeros: __________________________

Multiplicity: __________________________

For a real zero $x = c$:

- If the multiplicity is ________, the graph of $f(x)$ ________________the x-axis at the point ________.
- If the multiplicity is ________, the graph of $f(x)$ ________________the x-axis at the point ________.

(EX) Find the real zeros of the polynomial function, and their multiplicity. At each zero, indicate whether the function crosses or is tangent to the x-axis at that point.

(a) $P(x) = -2(x+5)^3(x-4)(x+1)^2$

(b) $P(x) = (x+4)^2(1-3x)(x^2+1)^3$

WeBWorK

Find the zeroes and multiplicities of the polynomial $f(x) = (x-7)^{10}(x^2-49)^6$.

The zeroes are $x =$ ______ (separate your answers by commas).

The zero $x =$ ______ has multiplicity ______ .

The zero $x =$ ______ has multiplicity ______ .

Sign Diagrams:

We can construct a sign diagram for a polynomial function, similar to the methods used for solving quadratic inequalities.

Suppose f is a polynomial function.

1. Find all zeros of f, and place them on a number line.
2. Select a test point in each interval from your number line.
3. Determine whether $f(x)$ is positive or negative when evaluated at each test point. This will determine whether the graph will lie above the x-axis (if $f(x) > 0$), or below the x-axis (if $f(x) < 0$).

Finally, we can compile all of this information for a polynomial function and construct a graph of the function.

To Graph a Polynomial Function:

1. Use the leading term test to determine the end behavior.
2. Find all zeros and their multiplicity by solving $P(x) = 0$. (All *real* zeros correspond to x-intercepts.)
3. Construct a sign diagram for the function.
4. Find the y-intercept by finding $P(0)$.

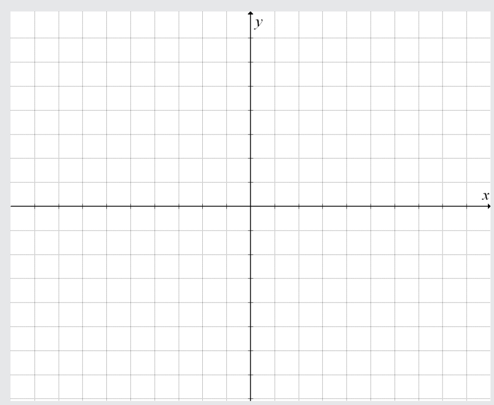
(EX) For each polynomial function, find the following information, and graph:

(a) $h(x) = x^3 - x^2 - x + 1$

Degree:

Leading coefficient:

End behavior:



Real zeros and their multiplicity:

Crosses or tangent to x-axis:

Sign Diagram:

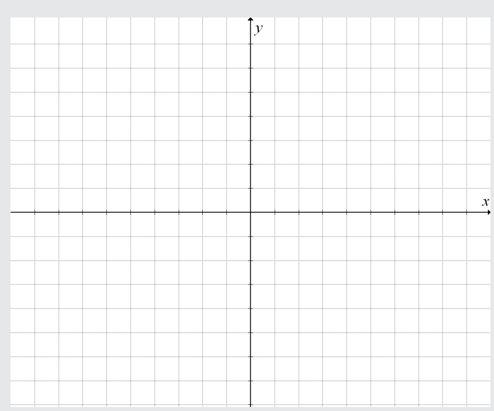
y-intercept:

(b) $g(x) = -x(x - 3)^2(x + 2)$

Degree:

Leading coefficient:

End behavior:



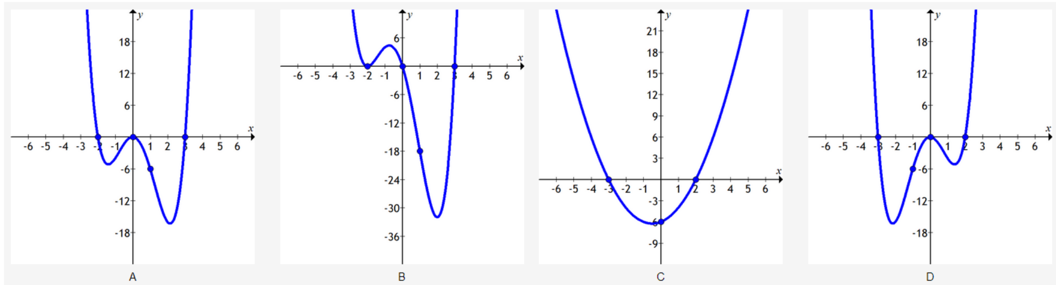
Real zeros and their multiplicity:

Crosses or tangent to x-axis:

Sign Diagram:

y-intercept:

WeBWorK



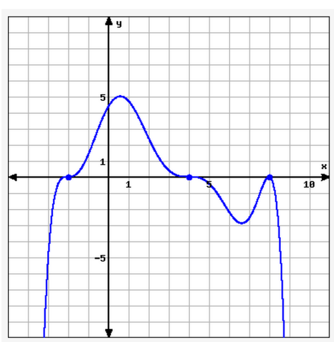
Given that

$$f(x) = x^4 + x^3 - 6x^2$$

determine which of the above graphs is the graph of f.

Correct Letter:

WeBWorK



Find the following information pertaining to the polynomial, $f(x)$, graphed above.

(a) The zeros of $f(x)$ are $x =$ (separate by commas) and

the zero $x =$ has multiplicity,

the zero $x =$ has multiplicity and,

the zero $x =$ has multiplicity.

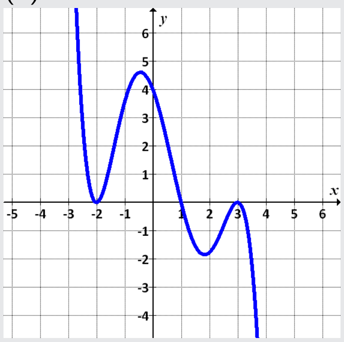
(b) The degree of $f(x)$ is .

(c) The leading coefficient of $f(x)$ is .

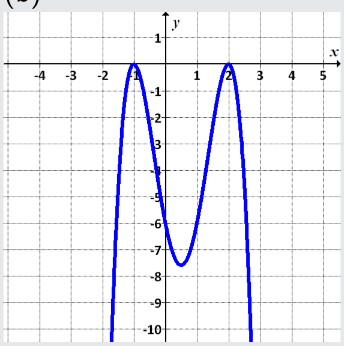
Help: Type **even** or **odd** for multiplicities and the degree. Type **pos** for positive or **neg** for negative for the leading coefficient. You must have the corresponding multiplicity correct for each zero in order to receive credit!

(EX) Determine the equation of the polynomial of minimum degree whose graph is shown:

(a)



(b)



Polynomial Inequalities: Our sign diagrams will allow us to solve polynomial inequalities using the same approach used for quadratic inequalities.

(EX) Solve:

(a) $x^3 + 9x^2 + 18x \geq 0$

(b) $x^3 + 5x^2 - 25x < 125$

WeBWorK

Solve the following inequality and write your answer using interval notation.

$$-7(u-5)(u+2)^2(u-9)>0$$

Answer:

4.1: Introduction to Rational Functions

Rational Functions:

A rational function is of the form $f(x) = \dfrac{p(x)}{q(x)}$, where p and q are polynomials, and $q(x) \neq 0$.

Since we typically will have variables in the denominator, this means that there will usually be values that have to be excluded from the domain of a rational function. (Recall that we must avoid all values for which the denominator equals zero.)

(EX) Find the domain of the rational function, in interval notation.

(a) $f(x) = \dfrac{-4x}{x^2 - 2x - 24}$

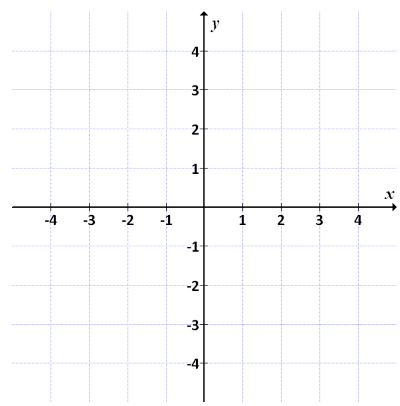
(b) $g(x) = \dfrac{2x^3 - 16}{3x^3 - 12x}$

On the graph of a rational function, different behavior can occur as we approach a value that is excluded from the domain. To see an example of this, let's examine the graph of one of the most basic rational functions:

Graph $f(x) = \dfrac{1}{x}$

Domain: ________________

Range: ________________



As you can see with this function, the value of $x = 0$ that is excluded from the domain ends up being an asymptote for the graph of the function. Values excluded from the domain of a rational function will either be a ____________________________________ or a ________________.

Vertical Asymptotes: A vertical asymptote (VA) is a line ________, which the graph approaches, but never crosses.

Holes: If $p(x)$ and $q(x)$ share a common factor, then the graph of $f(x) = \dfrac{p(x)}{q(x)}$ will have a hole at the x-value which corresponds to the common factor. (The y-coordinate of this hole can be found by substituting the x-value into the reduced form of the rational function.)

(EX) Find the vertical asymptotes and holes on the graph of the rational function.

(a) $f(x) = \dfrac{-4x}{x^2 - 2x - 24}$ (b) $g(x) = \dfrac{2x^3 - 16}{3x^3 - 12x}$

Horizontal Asymptotes:

A horizontal asymptote (HA) is a line ________, that the graph approaches as $x \rightarrow$________. (The graph can cross a HA, but doesn't have to.) In other words, the asymptotes describe the end behavior of the graph, as $x \to \pm\infty$.

Finding Horizontal Asymptotes:

For a rational function

$$f(x) = \frac{p(x)}{q(x)} = \frac{cx^n + \dots}{dx^m + \dots}$$

where n and m are the degrees of $p(x)$ and $q(x)$, respectively:

- If ________, then ________________is a horizontal asymptote of the function f.
- If ________, then f has NO horizontal asymptotes.
- If ________, then ________________is a horizontal asymptote of the function f, where c and d are the __________________________of $p(x)$ and $q(x)$, respectively.

(eg) $f(x) = \dfrac{5x^2 + 3x + 1}{2x - 1}$, $g(x) = \dfrac{2x - 1}{5x^2 + 3x + 1}$, $h(x) = \dfrac{2x^2 - 1}{5x^2 + 3x + 1}$

(EX) Find the horizontal asymptotes of the graph of the function.

(a) $f(x) = \dfrac{-4x}{x^2 - 2x - 24}$ (b) $g(x) = \dfrac{2x^3 - 16}{3x^3 - 27x}$

Review - Finding Intercepts: Recall that we find the x-intercepts of a function by solving $f(x) = 0$, and we find the y-intercepts by evaluating $f(0)$.

(EX) Consider the rational function $f(x) = \dfrac{2x^2 - 11x + 5}{x^2 - 25}$. Find all VA, HA, holes, and intercepts.

WeBWorK

Given the rational function

$$f(x) = \frac{x^3 + 2x^2 + x}{x^2 + 3x + 2}$$

find all of the following.

(a) Domain:

(b) Vertical asymptote(s) (if any): $x =$

(c) Hole(s) (if any): (Write the answer as an ordered pair.)

(d) Horizontal asymptote (if it exists): $y =$

4.2: Graphs of Rational Functions

Graphs of Rational Functions:

To sketch the graph of a rational function, we will use the methods from the last section for finding asymptotes, holes, and intercepts, along with a sign diagram.

1. Reduce the function to lowest terms (if applicable). For any canceled factors, find the location of the hole in the graph.
2. Find the equations of all vertical and horizontal asymptotes (if any).
3. Find all x-intercepts (if any) by solving $f(x) = 0$.
4. Find the y-intercept (if any) by finding $f(0)$.
5. Use a sign diagram and plot additional points, as needed, to sketch the graph.

(EX) Graph. Clearly label all asymptotes, intercepts, and holes.

(a) $f(x) = -\dfrac{6}{x+5}$

Domain:

Vertical Asymptote(s):

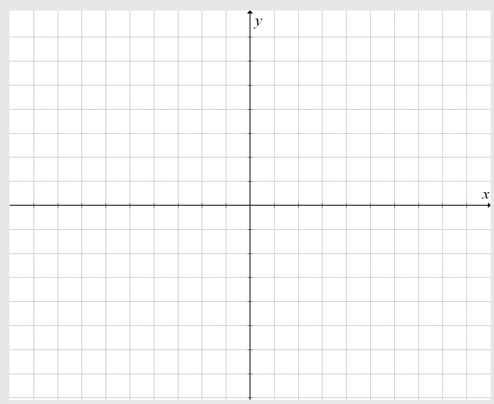
Horizontal Asymptote(s):

Hole(s):

x-intercept(s):

y-intercept:

Graph f on the axes given.



(b) $g(x) = \dfrac{2x-1}{(x-2)^2}$

Domain:

Vertical Asymptote(s):

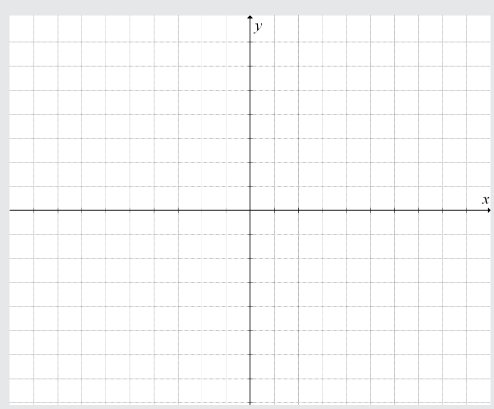
Horizontal Asymptote(s):

Hole(s):

x-intercept(s):

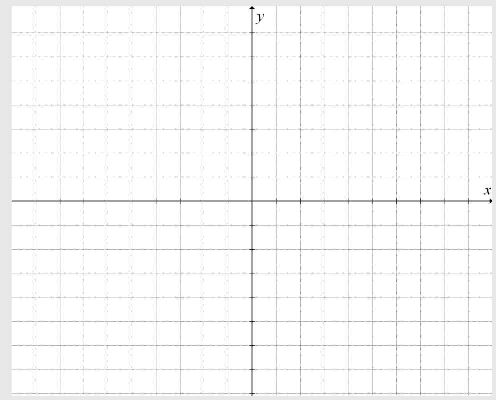
y-intercept:

Graph g on the axes given.



(c) $f(x) = \dfrac{x^2 - 4}{x^2 + 3x + 2}$

Domain:



Vertical Asymptote(s):

Horizontal Asymptote(s):

Hole(s):

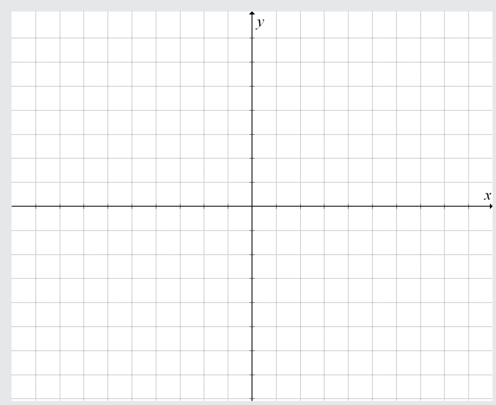
x-intercept(s):

y-intercept:

Graph f on the axes given.

(d) $g(x) = \dfrac{27}{x^2 + 9}$

Domain:



Vertical Asymptote(s):

Horizontal Asymptote(s):

Hole(s):

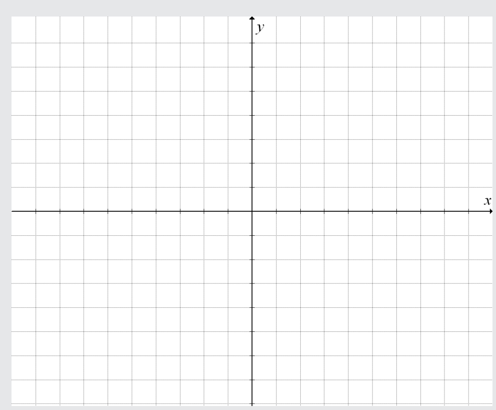
x-intercept(s):

y-intercept:

Graph g on the axes given.

(e) $h(x) = \dfrac{5x}{16 - x^2}$

Domain:



Vertical Asymptote(s):

Horizontal Asymptote(s):

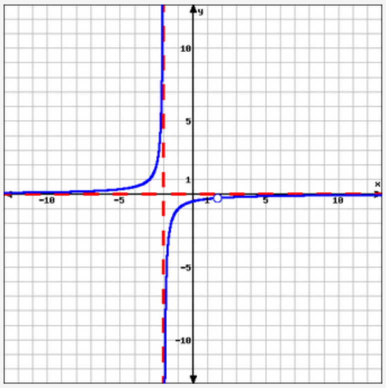
Hole(s):

x-intercept(s):

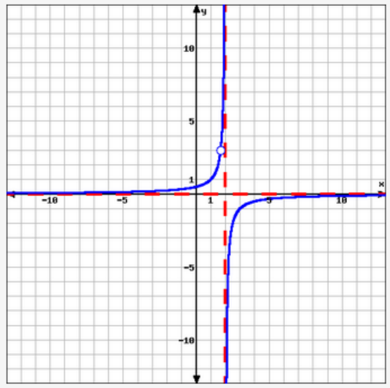
y-intercept:

Graph h on the axes given.

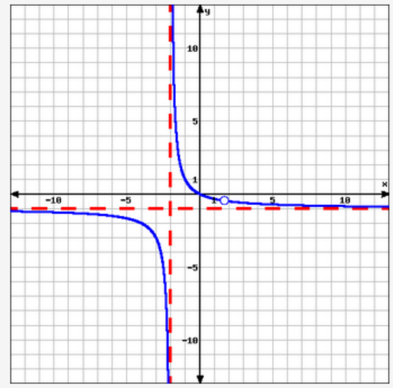
WeBWorK



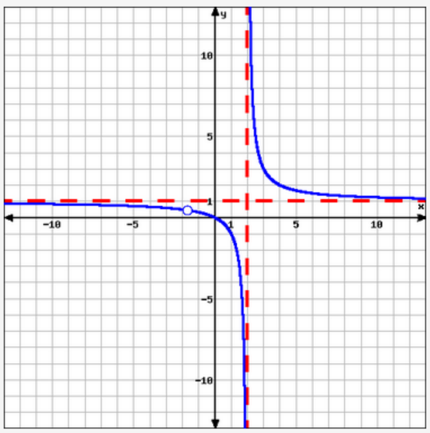
A



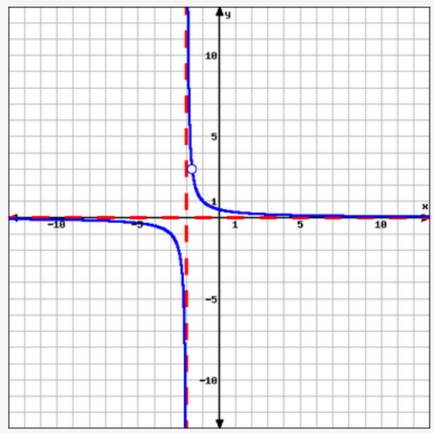
B



C



D



E

Determine the following information given that

$$f(x) = \frac{3x - 5}{-3x^2 - x + 10}$$

Separate multiple answers by commas. If an asymptote, hole, or intercept does not exist, write **none**. Note that you may click a graph to enlarge it.

(a) Horizontal Asymptote(s): $y =$

(b) Hole(s): (Type an ordered pair.)

(c) Vertical Asymptote(s): $x =$

(d) x-intercept(s): (Type an ordered pair.)

(e) y-intercept: (Type an ordered pair.)

(f) Letter Corresponding to Graph:

(EX) Find a rational function that satisfies the given conditions:

Vertical asymptote: $x = 4$, Horizontal asymptote: $y = -2$, y-intercept: $(0, 1)$

WeBWorK

Find a rational function $f(x)$ that satisfies the given conditions: The domain of f is $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$; there is a hole in the graph of f at the point $(1, -1)$, and the horizontal asymptote for f is $y = 0$.

A. $f(x) = \dfrac{x^2 - 4x + 3}{x^3 - x}$

B. $f(x) = \dfrac{x^3 - 4x^2 + 3x}{x^2 - 1}$

C. $f(x) = \dfrac{x^3 - 4x^2 + 3}{x^2 - 1}$

D. $f(x) = \dfrac{x^3 - 2x^2 - 3x}{x^2 - 1}$

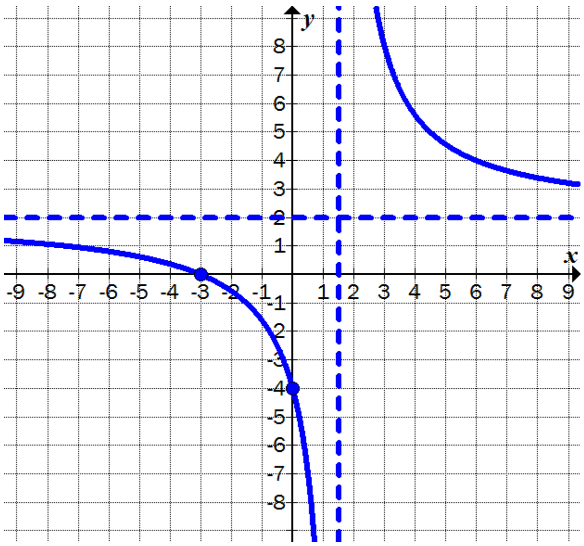
E. none of the above

Correct Letter:

4.3: Rational Inequalities

Rational Inequalities:

(eg) Given the graph of $f(x) = \dfrac{4x+12}{2x-3}$, construct a sign diagram and solve:



(a) $f(x) = 0$

(b) $f(x) \geq 0$

(c) $f(x) > 0$

(d) $f(x) \leq 0$

(e) $f(x) < 0$

Solving Rational Inequalities Algebraically:

1. Move everything to one side of the inequality, so that you are comparing with 0. (If necessary, get a common denominator.)

DO NOT CLEAR FRACTIONS!!

2. Find all critical points (all VA and x-intercepts on the graph of f). Also, find any holes.
3. Draw a number line and break the x-axis into intervals using the critical points.
4. Pick a test point in each interval and determine the sign $(+/-)$ of the function.
5. Remember to exclude all holes and VA in your final solution.

(EX) Solve. Use interval notation for your answer.

(a) $\dfrac{-2}{5-x} \geq 0$

(b) $\dfrac{x^2-2x+1}{x^2-5x-6} > 0$

(c) $\dfrac{x+2}{x-3} \leq 4$

WeBWorK

Solve the following inequality. Write your answer using interval notation.

$$\frac{w+5}{w+6} - \frac{w-10}{w-6} < 0$$

5.1: Function Composition

Composition of Functions: Suppose a student in a chemistry course needs to convert temperatures from Fahrenheit to Kelvin. The formulas that are available to her are:

$$\text{Convert Fahrenheit to Celsius: } C(t) = \frac{5}{9}(t-32)$$
$$\text{Convert Celsius to Kelvin: } K(t) = t + 273$$

Instead of constantly converting to Celsius, then to Kelvin, the student might want to develop one function that would work in a single step:

The new function is a composition of the original functions. In general, we define the composite function ________as:

____

where x is in the domain of ________and $g(x)$ is in the domain of ________.

(EX) Given $f(x) = \begin{cases} 2x+5 & \text{for } x<0 \\ 1-x^2 & \text{for } x \geq 0 \end{cases}$, and $g(x) = \dfrac{1}{x-1}$, find:

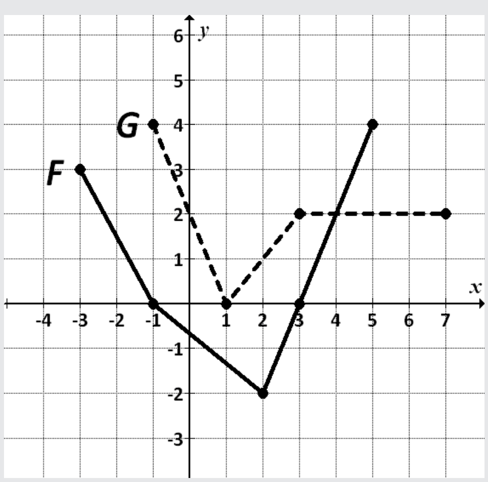
(a) $(f \circ g)(2)$

(b) $(f \circ g)(-3)$

(c) $(g \circ f)(0)$

(d) $(g \circ f)(-1)$

(EX) Given the graphs of F and G below, find the following:



(a) $(F \circ G)(3) =$ ________________

(b) $(G \circ F)(-1) =$ ________________

(c) $(F \circ F)(4) =$ ________________

(d) $(G \circ G)(0) =$ ________________

(EX) Given that $f(x) = \sqrt{x-9}$ and $g(x) = 4x^2$, find:

(a) $(f \circ g)(x)$ and its domain

(b) $(g \circ f)(x)$ and its domain

WeBWorK

Suppose that

$$f(x) = -2x + 4 \quad \text{and} \quad g(x) = 2x^2 + 2x + 3$$

Find the composition functions $(f \circ g)(x)$ and $(g \circ f)(x)$ and then give their domains in interval notation.

(a) $(f \circ g)(x) =$

(b) $(g \circ f)(x) =$

(c) Domain of $(f \circ g)(x)$:

(d) Domain of $(g \circ f)(x)$:

(EX) Given that $f(x) = \dfrac{1}{2-x}$ and $g(x) = \dfrac{1}{3x}$, find:

(a) $(f \circ g)(x)$ and its domain

(b) $(g \circ f)(x)$ and its domain

WeBWorK

Suppose that

$$f(x) = \frac{7}{3x+2} \quad \text{and} \quad g(x) = \frac{1}{x}.$$

Find the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$ and then give their domains in interval notation.

(a) $(f \circ g)(x) =$

(b) $(g \circ f)(x) =$

(c) Domain of $(f \circ g)(x)$:

(d) Domain of $(g \circ f)(x)$:

(EX) Find functions f and g such that $h(x) = (f \circ g)(x)$.

(a) $h(x) = \dfrac{1}{\sqrt{3x+7}}$

(b) $h(x) = (\sqrt{x} - 3)^4$

WeBWorK

Find a function $g(x)$ so that $h(x) = (f \circ g)(x)$ given that

$$f(x) = \frac{x+4}{x-2} \quad \text{and} \quad h(x) = \frac{x^6+4}{x^6-2}.$$

$g(x) =$

5.2: Inverse Functions

If we think of a function as a process, then in this section we are going to look for a new function that might reverse that process. Some processes (like getting dressed) are reversible, while others are not (like toasting bread).

A real-life example of reversing a process would be in giving directions between two places.

Directions from A to B:	Directions from B to A:
1. Leave point A and go north 1 mile.	1.
2. Turn right.	2.
3. Go two miles due east and arrive at point B.	3.

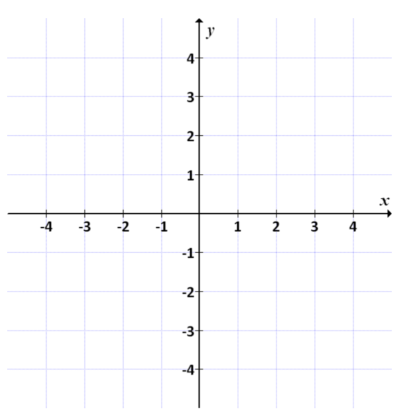
Now suppose that we have a mathematical process that we want to reverse:

Process for f:	Reverse Process for f:
1. Multiply by 2	1.
2. Subtract 3	2.
Formula for f:	Formula for g:

Check to see if g reverses f for a specific number ($x = 2$).

Check to see if g reverses f for any value of x.

Graph both f and g on the axes below.



Inverse Functions:

Suppose f and g are two functions such that:

1. $(g \circ f)(x) = x$ for all x in the domain of f **and**
2. $(f \circ g)(x) = x$ for all x in the domain of g

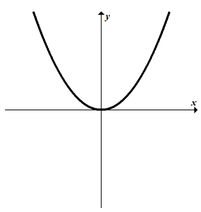
then f and g are said to be ______________________________ of each other. The functions f and g are said to be ______________________________.

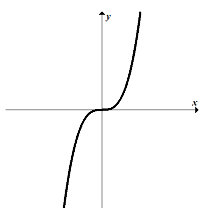
One-to-One Functions:

All of our properties of inverse functions hinge on the fact that a function must be invertible in order for its inverse function to exist. So, we need a way to identify when a function is invertible.

DEFINITION: A function f is called ___________________ (or invertible) if different inputs have different outputs.

Consider the graphs below.





Does each x-value (input) correspond to a unique y-value (output)?

Does each y-value (output) correspond to a unique x-value (input)?

The Horizontal Line Test:

A function f is ______________________________ if and only if no ______________________________ line intersects the graph of f more than once.

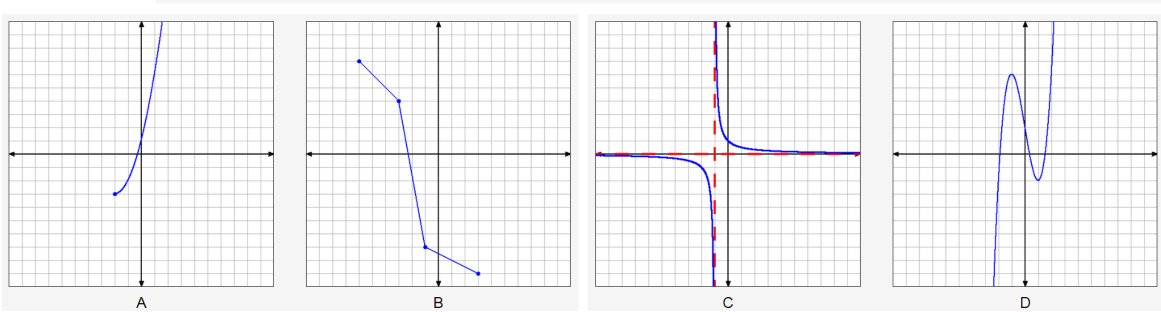
(EX) Which of the following functions are one-to-one? (HINT: Graph the function!)

(a) $f(x) = 4x^2 - 1$ (b) $f(x) = \sqrt{x+1}$ (c) $f(x) = |x - 2|$

WeBWorK

List the letters corresponding to the graphs below that represent one-to-one functions. Write "none" if none of the graphs represent a one-to-one function. If there are multiple letters, separate them by commas.

Correct Letters:



To find a formula for $f^{-1}(x)$ (when f is one-to-one):

1. Write $y = f(x)$.
2. Switch (reverse) x and y in the equation. (Write as $x = f(y)$.)
3. Solve for y to obtain $y = f^{-1}(x)$.

(EX) Given the one-to-one function $A = \{(4,5),(2,11),(-3,1),(6,2)\}$, find A^{-1}.

(EX) Given the one-to-one function below, find f^{-1}.

(a) $f(x) = 2 - \dfrac{3+5x}{4}$

(b) $f(x) = 2\sqrt[3]{x} - 3$

(c) $f(x) = (x+5)^3$

(d) $f(x) = \dfrac{2}{x+1}$

WeBWorK

Determine whether the following function is one-to-one on its domain. If it is one-to-one, find its inverse function $f^{-1}(x)$. If it is not-one-to-one, write "undefined" in the blank provided. You do not need to simplify your answer.

$$f(x) = \frac{2x-5}{6x+1}$$

$f^{-1}(x) =$ ______

Properties of Inverse Functions:

Suppose f is an invertible function.

- There is exactly one inverse function for f, denoted _________.
- $(f \circ f^{-1})(x) = f(f^{-1}(x)) =$ _________ for all x in the domain of _________.
- $(f^{-1} \circ f)(x) = f^{-1}(f(x)) =$ _________ for all x in the domain of _________.
- The Domain of $f =$ ____________________________.
- The Range of $f =$ ____________________________.
- The graph of f^{-1} is the reflection of the graph of f across the line __________________.
- $f(a) = b$ if and only if __________________.
- The point (a, b) is on the graph of f if and only if the point __________________ is on the graph of f^{-1}.

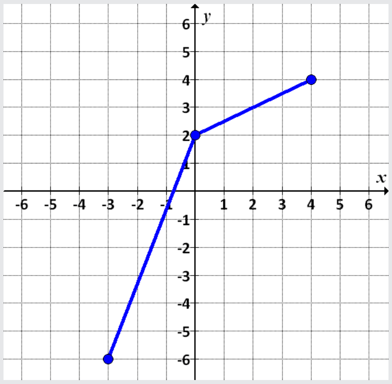
(EX) Use composition of functions to verify that f^{-1} is correct.

$$f(x) = \frac{x+6}{x}, \quad f^{-1}(x) = \frac{6}{x-1}$$

(EX) Using the function in the previous example, find $f^{-1}(f(100))$.

(EX) Given that f is one-to-one, and $f(5) = 11$, find a point on the graph of f^{-1}.

(EX) Given the graph of a one-to-one function below, find the following:



(a) $f(-3) =$________

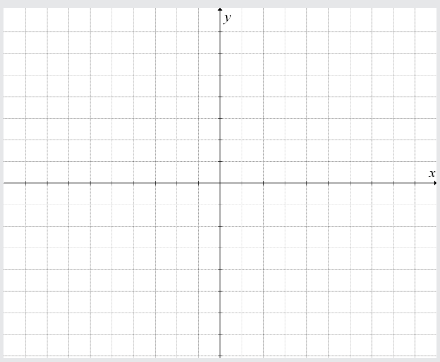
(b) $f^{-1}(-6) =$________

(c) $f^{-1}(2) =$________

(d) Sketch the graph of f^{-1}.

(EX) Given $f(x) = \dfrac{x-1}{x+2}$, find the following:

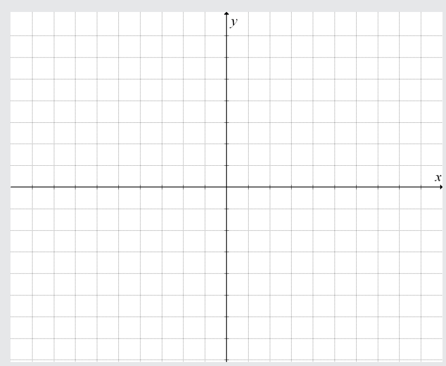
(a) Sketch $f(x)$. Is it one-to-one?



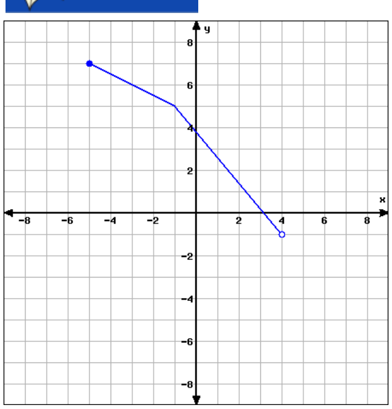
(b) Find a formula for $f^{-1}(x)$

(c) State the domain and range of both f and f^{-1}

(d) Sketch $f^{-1}(x)$.



WeBWorK



The graph of $f(x)$ is given above. Determine the domain and range of $f^{-1}(x)$ using interval notation.

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

6.1: Introduction to Exponential and Logarithmic Functions

Exponential Functions:

An exponential function is of the form $f(x) =$ __________, where $x \in \mathbb{R}$, $b > 0$, $b \neq 1$.

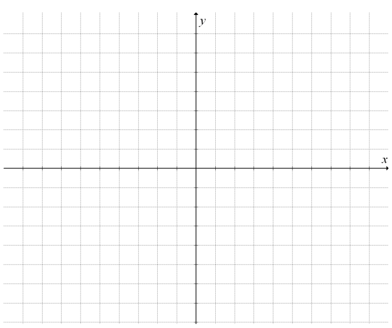
The graph of an exponential function will vary depending on the value of b:

Properties of Exponential Functions

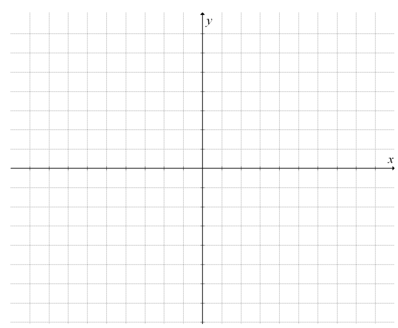
Suppose $f(x) = b^x$.

- The domain of f is ____________________, and the range of f is ____________________.
- The points ____________________ and ____________________ are on the graph of f.
- The line ____________________ is a horizontal asymptote of the graph of f.
- The function f is one-to-one, continuous, and smooth (topics that are discussed more in Calculus).

- If $b > 1$:
  - f is always ____________________
  - Called exponential ____________________
  - As $x \to -\infty$, $f(x) \to 0^+$
  - As $x \to \infty$, $f(x) \to \infty$
  - The graph of f resembles:

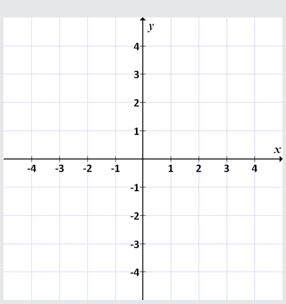


- If $0 < b < 1$:
  - f is always ____________________
  - Called exponential ____________________
  - As $x \to -\infty$, $f(x) \to \infty$
  - As $x \to \infty$, $f(x) \to 0^+$
  - The graph of f resembles:

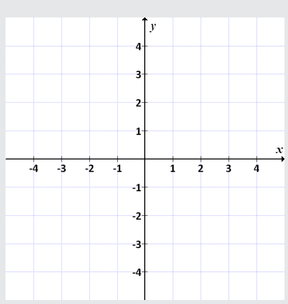


(EX) Graph each exponential function:

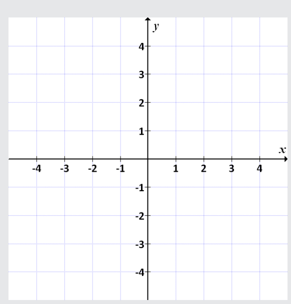
(a) $f(x) = 3^x$



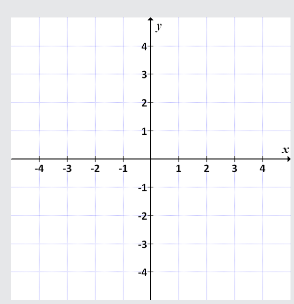
(b) $g(x) = (\frac{1}{5})^x$



(c) $h(x) = 6^x$



(d) $k(x) = (\frac{3}{4})^x$



Special Bases:

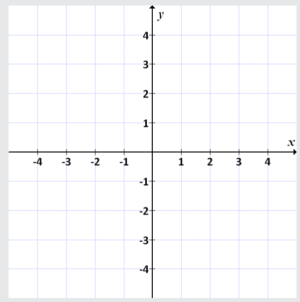
Of all the bases used for exponential functions, two occur most often in scientific circles:

- The ________________base: base 10
- The ________________base: base $e \approx 2.7182818284\ldots$ (This is called Euler's constant, and it is an irrational number whose origins are discussed in Section 6.5 in your text.)

Graphing Exponential Functions: We can use our transformations to help us to sketch exponential functions that have been modified:

(EX) For each exponential function, find the following information and graph:

(a) $f(x) = 2^{x+3} - 4$



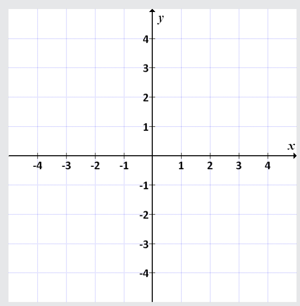
Domain:

Range:

y-intercept:

Asymptote:

(b) $g(x) = -(\frac{1}{2})^x + 1$



Domain:

Range:

y-intercept:

Asymptote:

WeBWorK

Let f be the exponential function defined by

$$f(x) = -e^{-x} + 5$$

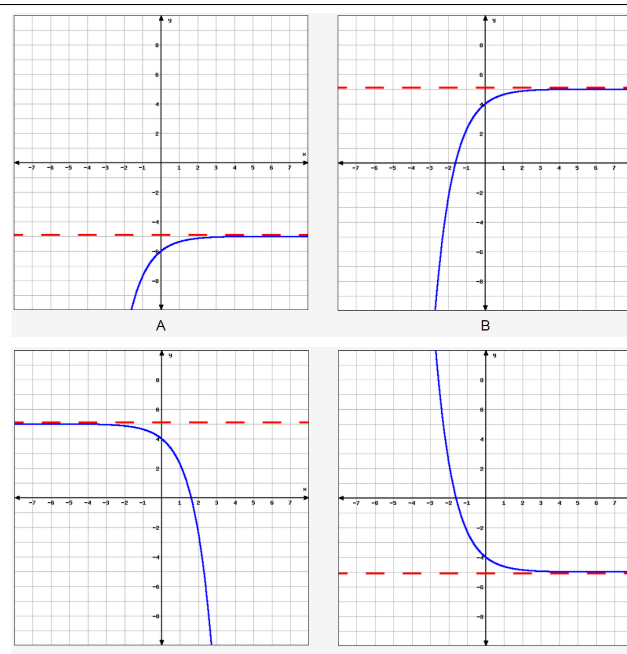
and determine the following information based on f.

(i) Horizontal Asymptote:

(ii) Range:

(iii) y-intercept:

(iv) Letter Corresponding to Graph:



Logarithmic Functions:

Since exponential functions are always one-to-one, we are able to find the inverse of any exponential function.

The inverse of the exponential function $f(x) = b^x$ is called the ________________________________,

and is written $f^{-1}(x) =$_______________. (Read 'log base b of x.')

$b^a = c$ if and only if $\log_b(c) = a$
That is, $\log_b(c)$ is the exponent you put on b to obtain c.

So, logarithms allow us to solve for the exponent in an expression, and we will often work with logarithms by relating them to an exponential form.

Exponential Form:	$2^x = 16$ $x =$________	$b^x = 1$ $x =$________	$a^x = a$ $x =$________	$3^x = \sqrt{3}$ $x =$________	$10^x = \frac{1}{1000}$ $x =$________
Logarithmic Form:					

Special Bases:

Using our special bases, we have notations for the corresponding logarithms:

- Base 10: The _______________logarithm: $\log_{10}(x) =$_______________
- Base e: The _______________logarithm: $\log_e(x) =$_______________

(EX) Convert each equation to the other form (logarithmic or exponential).

Exponential Form:	(a) $4^t = 7$	(b)	(c) $10^y = x^2 + 1$	(d)
Logarithmic Form:	(a)	(b) $\log_a M = -x$	(c)	(d) $\ln x = 4$

(EX) Evaluate without using a calculator:

(a) $\log_2(8)$ (b) $\log_3(\frac{1}{9})$ (c) $\log(\sqrt[3]{10})$

(d) $\ln(\frac{1}{\sqrt{e}})$ (e) $\log 10,000$ (f) $\log_3(\sqrt[5]{9})$

WeBWorK

Simplify the following quantities.

$\log_4(16) =$ ______

$\log_3(81) =$ ______

$\log_4\left(\frac{1}{16}\right) =$ ______

$\log(0.00001) =$ ______

$\ln(e^2) =$ ______

$\log_{18}(18^{25}) =$ ______

$\log_4(2) =$ ______

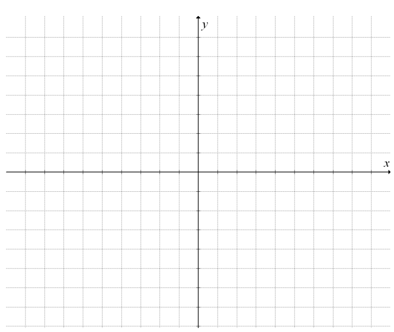
$\log_{31}(1) =$ ______

Properties of Logarithmic Functions

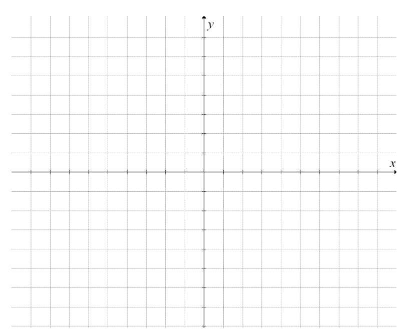
Suppose $f(x) = \log_b(x)$.

- The domain of f is ________________, and the range of f is ________________.
- $\log_b 1 =$ ________
- $\log_b b =$ ________
- The points ________________ and ________________ are on the graph of f.
- The line ________________ is a vertical asymptote of the graph of f.
- $\log_b(b^x) =$ ________ for all x.
- $b^{\log_b(x)} =$ ________ for all $x > 0$.
- The function f is one-to-one, continuous, and smooth (topics that are discussed more in Calculus).

- If $b > 1$:
  - f is always ________________
  - As $x \to 0^+$, $f(x) \to -\infty$
  - As $x \to \infty$, $f(x) \to \infty$
  - The graph of f resembles:



- If $0 < b < 1$:
  - f is always ________________
  - As $x \to 0^+$, $f(x) \to \infty$
  - As $x \to \infty$, $f(x) \to -\infty$
  - The graph of f resembles:



(EX) Evaluate without using a calculator:

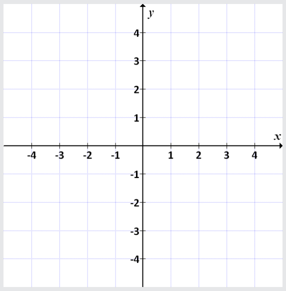
(a) $\log_b(b^3)$

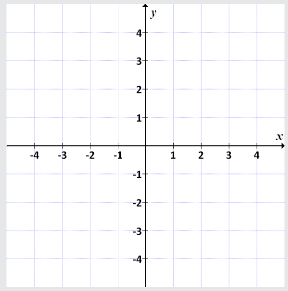
(b) $3^{\log_3(87)}$

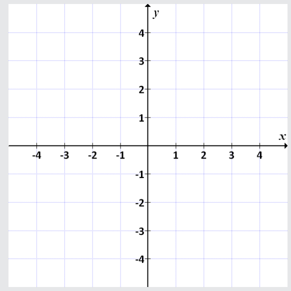
(c) $\log(10^{x-9})$

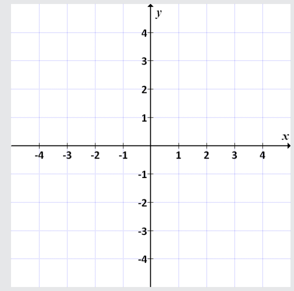
(EX) Graph each logarithmic function:

(a) $f(x) = \log_3(x)$ (b) $g(x) = \log_{1/5}(x)$ (c) $h(x) = \log(x)$ (d) $k(x) = \ln(x)$









(EX) For each logarithmic function, find the following information and graph:

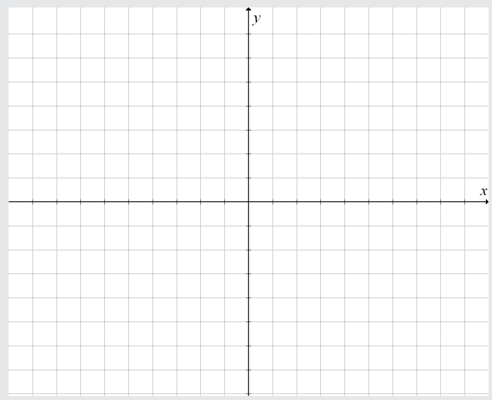
(a) $f(x) = \ln(x + 1)$

Domain:

Range:

y-intercept:

Asymptote:



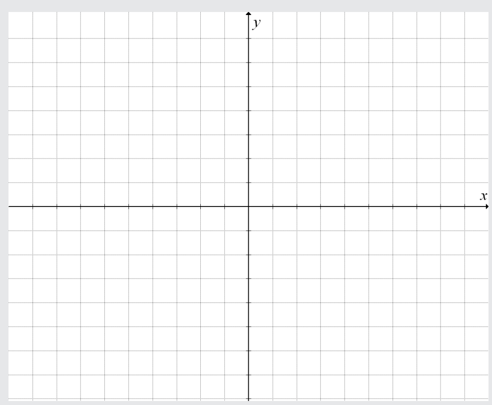
(b) $g(x) = -\log_3(x) - 2$

Domain:

Range:

y-intercept:

Asymptote:



(EX) Find the domain of the function (using interval notation).

(a) $f(x) = \log(4 - 3x)$

(b) $f(x) = \log_5(9 - x) + \log_5(x - 2)$

WeBWorK

Let f be the logarithmic function defined by

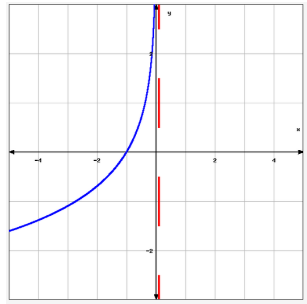
$$f(x) = -\ln(-x)$$

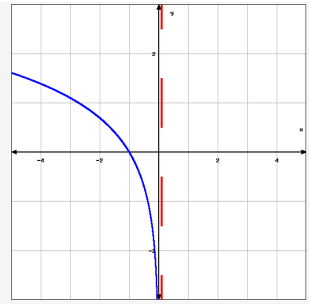
and determine the following information based on f. If such a value does not exist, type **none**.

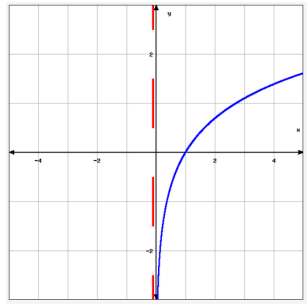
(i) Vertical Asymptote:

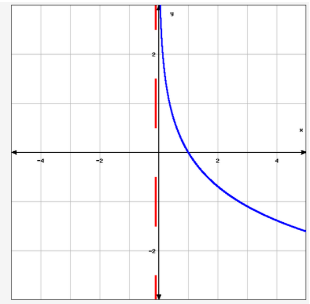
(ii) Domain:

(iii) Letter Corresponding to Graph:


A


B


C


D

6.2: Properties of Logarithms

We have already discussed the fact that exponential and logarithmic functions are inverses of one another. In this section we will dive further into the basic properties of exponential functions, and look at their analogous versions for logarithmic functions.

Properties of Exponential Functions

Suppose $f(x) = b^x$, $b > 0, b \neq 1$.

Let u and w be real numbers.

Product Rule: $b^u b^w =$ ________________.

Quotient Rule: $\dfrac{b^u}{b^w} =$ ________________.

Power Rule: $(b^u)^w =$ ________________.

Properties of Logarithmic Functions

Suppose $f(x) = \log_b(x)$, $b > 0, b \neq 1$.

Let $u > 0$ and $w > 0$ be real numbers.

Product Rule: $\log_b(uw) =$ ________________.

Quotient Rule: $\log_b\left(\dfrac{u}{w}\right) =$ ________________.

Power Rule: $\log_b(u^w) =$ ________________.

(EX) Expand and express in terms of sums and differences of logarithms; simplify. Assume when necessary that all quantities represent positive real numbers.

(a) $\ln 7e^4$

(b) $\log \dfrac{xy}{10}$

(c) $\log_2 \sqrt[5]{\dfrac{4x^3}{5(y+1)}}$

WeBWorK

Write the following as a sum of logarithms:

$$\log\left(\sqrt{\frac{x^5y^6}{z^4}}\right) = \square \log(x) + \square \log(y) + \square \log(z)$$

(EX) Express as a single logarithm and, if possible, simplify.

(a) $\log x + \log 5$

(b) $\ln 33 - \ln 11 + 2\ln 4$

(c) $2\log_a x - 4\log_a y + \frac{1}{3}\log_a z$

(d) $\log_3 4 + \log_3 15 - \log_3 180$

WeBWorK

Use the properties of logarithms to write the expression as a single logarithm.

(a) If

$$\log(x) - \frac{1}{9}\log(z) + \frac{1}{9}\log(y) = \log(A),$$

then $A =$

(b) If

$$-\frac{1}{9}\ln(x) - \frac{1}{9}\ln(y) + \frac{1}{9}\ln(z) = \ln(B),$$

then $B =$

(EX) Given that $\log_b 3 \approx 0.6$, $\log_b 5 \approx 0.8$, and $\log_b 13 \approx 1.3$, find approximations for the following, if possible:

(a) $\log_b 15$

(b) $\log_b 169$

(c) $\log_b \frac{39}{5}$

(d) $\log_b 8$

Change of Base Formula: To change a logarithm from a base a to a base b, use the formula:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

(EX) Use the change of base formula to rewrite the logarithm in the indicated base:

(a) $\log_5(x+2)$ to base e

(b) $\log_3(12x^2+7)$ to base 10

(EX) Use the change of base formula to write as a single logarithm, and simplify.

(a) $\dfrac{\log 16}{\log 2}$

(b) $\dfrac{\log_3 \frac{1}{25}}{\log_3 5}$

(c) $\log_3(x+1) + \log_9(x)$

WeBWorK

Suppose that

$$\log_{67}(3) = w \text{ and } \log_{67}(8) = m.$$

Then, write the following quantity in terms of w and m.

$\log_8(3) =$ ______ .

(EX) Simplify:

(a) $\log 10^{2x}$

(b) $\ln(ex)$

(c) $10^{\log x^3}$

(d) $e^{2\ln x}$

(e) $\log_5 \sqrt{5^3}$

(f) $\log_3 9x^2$

(EX) True or False?

(a) $\log 3x + \log 2x = \log 5x$

(b) $\ln 2x^2 - \ln x = \ln 2x$

(c) $\log x^2 + \log x^3 = 5 \log x$

(d) $(\ln x)^2 = 2 \ln x$

(e) $\dfrac{\log x}{\log 2} = \log_2 x$

(f) $\ln x - \ln y = \dfrac{\ln x}{\ln y}$

WeBWorK

True or False. Assume all variables are positive, with $a, b \neq 0, 1$.

select **1.** $6^{\log_6 x} = x$

select **2.** $\log_b b = 0$

select **3.** $\log_b(x \cdot y) = \log_b x + \log_b y$

select **4.** $\log_b(x^r) = (\log_b x)^r$

select **5.** $\log_b b = b$

6.3: Exponential Equations

Solving Exponential Equations

1. Isolate the exponential function.

    Type I. COMMON BASE: Write both sides with a common base and equate the exponents.

    One-to-One Property: $b^u = b^w$ if and only if $u = w$.

    Type II: DIFFERENT BASES: Isolate the exponential, and take the natural log of both sides of the equation. Use the Power Rule to bring the variable out of the exponent.

2. If neither option above is successful, try to write the equation as a modified quadratic.

Exponential Equations - Type I (Common Base)

(EX) Solve:

(a) $3^{5x} = 9$

(b) $\dfrac{2^{x^2}}{4^{2x}} = \dfrac{1}{8}$

WeBWorK

Solve the equation for s:

$$\left(C^{2s+2}\right)^3 \cdot C^{6s-3} = C^9$$

$s =$

Help: Try matching bases to solve the equation.

Exponential Equations - Type II (Different Bases)

(EX) Solve:

(a) $3^{7x} = 11$

(b) $12 - e^{4x} = 7$

(c) $2^x = 5^{x+1}$

(d) $40 = \dfrac{80}{10 - e^{-0.7t}}$

WeBWorK

Solve the equation for x:

$$11^{9x-2} = 5^x$$

$x =$

Help: Give an exact answer. If necessary, type **log(b,x)** to enter $\log_b(x)$ as an answer. For example, type **log(3,5)** to enter $\log_3(5)$. You may also use **ln(15)** for $\ln(15)$, and **log10(20)** for $\log(20)$. Enter your answers as a comma separated list if there is more than one correct answer. Type **no solution** if the equation has no solution.

Exponential Equations - Modified Quadratic

(EX) Solve:

(a) $3e^{2t} + e^t - 2 = 0$

(b) $e^x + e^{-x} = 2$

(EX) For each one-to-one function below, find the x- and y-intercepts, and the inverse function.

(a) $f(x) = 2^{5x} + 1$

(b) $g(x) = 4 - 2e^{7x+1}$

WeBWorK

Find the x- and y-intercepts of $f(x) = 8^{-3x} - 2$. Write **none** if such a point does not exist.

x-intercept:

y-intercept:

Help: Use **log(b,x)** to enter logarithms that are not base e nor base 10. For example, type **log(3,5)** to enter $\log_3(5)$. Use **ln(15)** for $\ln(15)$, and **log10(20)** for $\log(20)$. Click here for help entering points as an answer.

WeBWorK

Find the inverse function of $f(x) = 10^{-4x+3} + 8$.

$f^{-1}(x) =$

Help: Use **log(b,x)** to enter logarithms that are not base e nor base 10. For example, type **log(3,5)** to enter $\log_3(5)$. Use **ln(15)** for $\ln(15)$, and **log10(20)** for $\log(20)$. Click here for help entering formulas as an answer.

6.4: Logarithmic Equations

Solving Logarithmic Equations

1. Identify the type of logarithmic equation you are solving:
   - Type I. SINGLE LOGARITHM: Isolate the logarithm and solve by rewriting in exponential form.

     $$\log_b a = c \text{ if and only if } b^c = a$$

   - Type II: MULTIPLE LOGARITHMS:
     (a) Equations with a constant term: Combine all logs into a single log on one side of the equation. Then solve by rewriting in exponential form (see above).
     (b) Equations without a constant term: Combine all logs into a single log on both sides of the equation. Solve by using the property below:

     One-to-One Property: $\log_b(u) = \log_b(w)$ if and only if $u = w$.

2. If neither option above is successful, try to write the equation as a modified quadratic.
3. CHECK YOUR ANSWERS for extraneous solutions. (Remember that each quantity inside a logarithm must be positive.)

Logarithmic Equations - Type I

(EX) Solve:

(a) $\log_4 x = 2$

(b) $\log(x^2 - 3x) = 1$

(c) $2\ln(x+1) - 4 = 7$

Logarithmic Equations - Type II - Equations with a constant term.

(EX) Solve:

(a) $\log x + \log(x-9) = 1$

(b) $\log_3(2x+1) = 1 - \log_3(x-2)$

WeBWorK

Solve the equation for y:

$$\log_5(y) - \log_5(y-6) = 3$$

$y =$

Help: Enter your answers as a comma separated list if there is more than one correct answer. Type **no solution** if the equation has no solution.

Logarithmic Equations - Type II - Equations without a constant term.

(EX) Solve:

(a) $\log(x+8) - \log(x+1) = \log 6$

(b) $\log_7(x+1) + \log_7(2x-1) = \log_7 5$

WeBWorK

Solve the equation for v:

$$\ln(v+6)+\ln(v-3)=\ln(7)$$

$v =$

Help: Enter your answers as a comma separated list if there is more than one correct answer. Type **no solution** if the equation has no solution. If necessary, type **sqrt()** to enter a square root. For example, to enter $\sqrt{7x}$, you would type your answer as sqrt(7x). You must put parenthesis around everything you want to take the square root of!!

Logarithmic Equations - Other - Equations requiring a change of base, or given in modified quadratic form.

(EX) Solve:

(a) $\log_5(x)=\log_{\frac{1}{5}}(x)+3$

(b) $(\log(x))^2=2\log(x)+15$

WeBWorK

Solve the equation for p:

$$\log_3(p)=\log_{1/3}(p)+8$$

$p =$

Help: Enter your answers as a comma separated list if there is more than one correct answer. Type **no solution** if the equation has no solution.

(EX) For each one-to-one function below, find the x- and y-intercepts, and the inverse function.

(a) $h(x) = \log_4(16 - 3x) + 1$

(b) $k(x) = 2\log(x - 1)$

WeBWorK

Find the x- and y-intercepts of $f(x) = 10\log_9(3x + 10) - 20$. Write **none** if such a point does not exist.

x-intercept:

y-intercept:

Help: Use **log(b,x)** to enter logarithms that are not base e nor base 10. For example, type **log(3,5)** to enter $\log_3(5)$. Use **ln(15)** for $\ln(15)$, and **log10(20)** for $\log(20)$. Click here for help entering points as an answer.

WeBWorK

Find the inverse function of $f(x) = 2\ln(3-2x) - 4$.

$f^{-1}(x) =$

Help: Use **log(b,x)** to enter logarithms that are not base e nor base 10. For example, type **log(3,5)** to enter $\log_3(5)$. Use **ln(15)** for $\ln(15)$, and **log10(20)** for $\log(20)$. Click here for help entering formulas as an answer.