Practice Problem Set 7

Throughout this assignment we assume that X is a nonempty ordered set.

(1) Let $\mathbb{F}$ be a field and suppose $x, y \in \mathbb{F}$. Prove the following statements:

  (a) If $x, y \in \mathbb{F}$ with $x \neq 0$, and $x \cdot y = x$, then $y = 1$.

  (b) If $x, y \in \mathbb{F}$ with $x \neq 0$, and $x \cdot y = 1$ then $y = x^{-1}$.

(2) Let $\mathbb{F}$ be an ordered field and suppose $x, y, z \in \mathbb{F}$. Prove the following statements:

  (a) If $x < 0$ and $y < z$, then $x \cdot y > x \cdot z$.

  (b) If $x < 0$ and $y > 0$, then $x \cdot y < 0$.

(3) Suppose $A \subseteq X$ is a nonempty set which is bounded from above in X. Show that if the supremum of A exists in X then it is unique. (*The uniqueness of the supremum justifies our use of the notation* $\sup(A)$.) *Note: The infimum of a set is also unique.*

(4) Let $A := \left\{ \dfrac{x}{x+1} : x \in \mathbb{Q},\ x > 0 \right\}$. Show that $\inf(A) = 0$.

(5) If possible, give an example of:

  (a) a set $A \subseteq \mathbb{Q}$ such that $\sup(A) = 3$ and $3 \notin A$.

  (b) a set $A \subseteq \mathbb{N}$ such that $\sup(A) = 3$ and $3 \notin A$.

  (c) a set $A \subseteq \mathbb{N}$ such that $\sup(A) > 3$ and $3 \notin A$.

(6) Let $A, B \subseteq X$ be two nonempty sets which are bounded from above in X. Suppose that $A \cap B \neq \emptyset$ and that $\sup(A), \sup(B) \in X$ exist.

  (a) Show that $\sup(A \cap B) \leq \min\{\sup(A), \sup(B)\}$.

  (b) Give an example of two nonempty sets A and B (as above) where $\sup(A \cap B) < \sup(A)$ and $\sup(A \cap B) < \sup(B)$.

  The problem above tells us that, although we know $\sup(A \cup B) = \max\{\sup(A), \sup(B)\}$ *(from HW), in general we do not have* $\sup(A \cap B) = \min\{\sup(A), \sup(B)\}$.