Practice Problem Set 5

(1) Let $f : A \to B$ be a function. Prove the following statements.

(a) If $C \subseteq D \subseteq A$ then $f(C) \subseteq f(D)$.

(b) If $C \subseteq D \subseteq B$ then $f^{-1}(C) \subseteq f^{-1}(D)$.

(c) If $y, z \in B$ with $y \neq z$ then $f^{-1}(\{y\}) \cap f^{-1}(\{z\}) = \emptyset$.

(2) Let $f : A \to B$ and $g : B \to C$ be two functions. Prove or disprove (with a counterexample) the following statements.

(a) If f and g are injective then $g \circ f : A \to C$ is injective.

(b) If $g \circ f : A \to C$ is injective then f is injective.

(c) If $g \circ f : A \to C$ is injective then g is injective.

(3) Prove that if $f : A \to B$ is an injective function then $f|_E : E \to B$ is injective for every nonempty subset $E \subseteq A$.

(4) Let $f : A \to B$ be a given function. Prove the following statements.

(a) f is surjective if and only if $f(A) = B$.

(b) If $f : A \to B$ is injective then $f : A \to f(A)$ is bijective.

(5) Given $n \in \mathbb{N}$, a set A has n elements if and only if there exists a bijective function $f : A \to \mathbb{N}_n$.

(6) A set A is finite if and only if there is a finite set B and a bijective mapping $f : A \to B$.