Practice Problem Set 1

(1) Suppose A, B, C, and X are sets such that $A, B, C \subseteq X$. Prove the following statements.

(a) $A \cap (A \cup B) = A$ and $A \cup (A \cap B) = A$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(c) [DeMorgan's Laws] $(A \cap B)^c = A^c \cup B^c$ and $(A \cup B)^c = A^c \cap B^c$

(d) $A \subseteq A \cup B$

(e) $A \cap B \subseteq B$

(f) If $A \subseteq B \cup C$ and $A \cap B = \emptyset$, then $A \subseteq C$.

(g) $C \subseteq A \cap B$ if and only if $C \subseteq A$ and $C \subseteq B$.

(h) $A \cup B \subseteq C$ if and only if $A \subseteq C$ and $B \subseteq C$.

(i) $A \cup \emptyset = A$

(j) $A \cap \emptyset = \emptyset$

(k) $A \cap X = A$

(l) $A \cup X = X$

(m) $A \cap A^c = \emptyset$

(n) $A \cup A^c = X$

(o) $\left(A^c\right)^c = A$

(p) $A = \emptyset$ if and only if $A^c = X$

(q) $A \cup B = A \cup (B \setminus A)$ and $A \cap (B \setminus A) = \emptyset$

(r) If $B \subset C$ then $A \setminus C \subseteq A \setminus B$

(2) For each of the following, give an example of sets A, B, and C satisfying the given conditions. Be sure to show that your example satisfies the necessary demands.

(a) $A \subseteq B$, $B \not\subseteq C$, and $A \subseteq C$

(b) $A \subseteq B$, $B \not\subseteq C$, and $A \not\subseteq C$

(3) Proofs involving integers. You may assume the basic operations and properties of integers.

(a) Suppose $n \in \mathbb{Z}$. If n is odd then n^2 is odd.

(b) Suppose $n, m \in \mathbb{Z}$. If n is even and m is odd then $m + n$ is odd.

(c) Suppose $n, m \in \mathbb{Z}$. If n and m are odd then the product nm is odd.

(d) Suppose $n, m \in \mathbb{Z}$. If n is even then the product nm is even.

(e) Suppose $n, m \in \mathbb{Z}$. If n and m have the same parity, i.e., if n and m are either both even or both odd, then the sum $n + m$ is even. (Try cases)

(f) Suppose $n, m \in \mathbb{Z}$. If n and m have opposite parity then the product nm is even.

(4) Suppose A, B, and C are sets. Show that if $A \subseteq B$ then $A \cup C \subseteq B \cup C$. What is the converse statement of this implication? Is it true or false? Prove if true and find a counterexample if false.