Math 413: Introduction to Theoretical Mathematics

Lecturer: Dr. Ryan Alvarado
E-mail: alvarado@pitt.edu
Office: 422 Thackeray Hall

Lecture Times: MWF 1:00PM - 1:50PM
Lecture Location: 103 Allen Hall

About the course

Course Description: This course covers the foundations of theoretical mathematics and analysis. The principal topics of the course include fundamentals of logic, sets, functions, number systems, order completeness of the real numbers and its consequences, and convergence of sequences and series of real numbers.

Core Topics:

1. Logic, quantifiers and proof techniques. Naive set theory, Russell's Paradox and the ZF axioms.
2. Elementary properties of natural numbers (Peano Property); Principle of Mathematical Induction.
3. Functions (compositions, injections, surjections, and bijections)
4. Finite sets; Pigeon Hole Principle. Infinite sets (countable and uncountable). Cardinality of sets.
5. Number systems. Countability of the natural numbers, integers, and rationals.
6. Construction of the real number system: axiomatic introduction to the ordered fields, least upper bounded property, and Dedekind cuts.
7. Properties of the real numbers: Archimedean Property; density of the rational and irrational numbers in the real numbers. Uncountability of the real numbers.
8. Sequences: limits, limit laws, Monotone Convergence Theorem, Bolzano-Weierstrass Theorem. Cauchy sequences and the Cauchy completeness of the real numbers.
9. An introduction to series: geometric series, convergence tests, alternating series, conditional convergence and Riemann rearrangement theorem.

Prerequisites: Successful completion of Math 0230 (Calculus II) or equivalent is required to follow this course. If you do not feel comfortable with the prerequisite material, please contact the instructor in the beginning of the course.

Course Objectives: Broadly speaking, the goal of this course is to prepare students for success in more advanced studies of abstract Mathematics by formalizing their ability to read, analyze, and communicate Mathematics in both written and oral formats. More specifically, upon a successful completion students will be able to:

- [Critical Thinking]
  Read and analyze a given mathematical proof so as it determine its validity.
- [Written Communication]
  Provide clear, mathematically rigorous arguments for the validity of a given theorem.
- [Oral Communication]
  Present and defend a written proof to a group of their peers.

Required Text: Lectures will be based on my own personal notes which complement the book Basic Analysis by Jiří Lebl. A free PDF copy of this text can be found at:

http://calculus.math.pitt.edu/books/pittanal.pdf.

Supplemental Texts: There are several books in the current literature addressing the topic of introducing students to mathematical proofs. Each text has a unique approach to the topic so do not hesitate to consult books outside of the require text. Here are some books that have good reviews:

- Introduction to Real Analysis by Robert G. Bartle and Donald R. Sherbert
- How to Read and Do Proofs: An Introduction to Mathematical Thought Processes by Daniel Solow
- Transition to Higher Mathematics: Structure and Proof by Bob A. Dumas and John E. McCarthy
- Mathematical Thinking and Writing: A Transition to Higher Mathematics by Randall Maddox

Course Requirements

Recitations: Twice a week you will meet with your TA. During this time you will be able to ask your TA to clarify topics that have been covered in class. You will also be assigned problem sets to work on in small groups. You will be required to individually present and defend your work to your peers at the board (see presentations, below).

Grading Scale: The nonnegotiable grading scale for this course is as follows:

Letter Grade	Minimum Score	Letter Grade	Minimum Score
A+	97%	C+	77%
A	93%	C	73%
A-	90%	C-	70%
B+	87%	D+	67%
B	83%	D	60%
B	80%	F	0%

NOTE: Course grades will not be curved.

Distribution of Points: Your course grade will be determined as follows:

Two In-Class Midterm Exams – 40% (20% each)
In-Class Final Exam (Comprehensive) – 20%
Written Homework – 15%
In-Class Presentations – 10%
Proof Portfolio – 10%
Participation – 5%

In-Class Midterm Exam Dates (Tentative):

Exam 1: The week of October 2nd
Exam 2: The week of November 6th

Final Exam Date: The final exam will take place in 103 Allen Hall on Monday, December 11th from 4:00PM-5:50PM.

Make-Up Exam Policy: Missed exams cannot be made up unless missed due to a conflict with an official university sponsored event, and I have has been notified *at least two weeks in advance*. Verifiable doctor's excuse will be considered on a case-by-case basis. Consideration may be given in the event of an extreme emergency if verifiable is provided.

Written Homework Assignments: You will be assigned written homework weekly. Homework is collected at the beginning of class on the due date and will be graded by the course's teaching assistant. For each assignment, only five problems will be graded for correctness. I encourage each of you to work in collaboration with your peers but all written work must finally be in your own expression.

Late-Homework Policy: I will accept late homework with 25% penalty for each late day, for up to two days. For example, if you are 1-24 hours late then your score cannot exceed 75% credit. There are no exceptions to this policy.

Proof Portfolio: You will be required to make a portfolio showing your progress over the course of the semester. The portfolio will consist of five entries, each based on a problem (from a homework set, worksheet, or exam) which was difficult for you. In most cases this should mean that your initial solution was incorrect as a result of a conceptual error, not just a simple computational error. Each entry should contain:

- Your original solution.
- An explanation of what was wrong with your original solution.
- A clearly-written, correct solution.

Please see me if you do not feel as though you have at least five conceptually incorrect proofs to submit for the portfolio. The portfolio will also have a 1-2 page essay addressing what you have learned about proof writing and the role of proofs in mathematics over the course of the semester. Guidelines and a grading rubric will be given out during the semester.

In-Class Presentations: Each week, one recitation will be set aside for students to present their proofs to a given set of problems. Every student in the class will be asked to present more than once during the course of the semester. You will be assessed on both the content of their proof and on the quality of the presentation. Specific guidelines for presentations will be given to you by your instructor.

Participation: Participation is key to both your learning and your peers learning. You will be required to actively participate in classroom discussions and when problem solving in groups. When you are observing a presentation, it is your responsibility to follow the logic of the solution and verify that it is correct for yourself. You may be asked during class to re-explain an argument that you just heard. If you cannot follow the reasoning, it is your responsibility to ask a question of the student presenting.

Getting Help

Tutoring: Walk in tutoring is available in the Calculus/Engineering Lab and in the Math Assistance Center (MAC) in room 215 of the O'Hara Student Center. Tutoring hours will be posted outside the lab and the MAC, as well as on the web at http://calculus.math.pitt.edu.

Disability Resource Services: If you have a disability for which you are or may be requesting an accommodation, you are encouraged to contact both your instructor and the Office of Disability Resources and Services, 216 William Pitt Union (412)-624-7890 as early as possible in the term.

Academic Integrity

Cheating/plagiarism will not be tolerated. Students suspected of violating the University of Pittsburgh Policy on Academic Integrity will incur a minimum sanction of a zero score for the quiz, exam or paper in question. Additional sanctions may be imposed, depending on the severity of the infraction.

Statement on Classroom Recording

To ensure the free and open discussion of ideas, students may not record classroom lectures, discussion and/or activities without the advance written permission from me, and any such recording properly approved in advance can be used solely for the students own private use.