HOMEWORK IX

Due: Monday, December 11th, 2017.

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1. Let $A \subseteq \mathbb{R}$ be a nonempty set which is bounded from above in $\mathbb{R}$ (so that $\sup(A) \in \mathbb{R}$ exists). Prove that there exist a nondecreasing sequence $(a_n)_{n=1}^{\infty}$ of points in A such that $\lim_{n\to\infty} a_n = \sup(A)$.
2. Recall that $\mathbb{Q}$ is dense in $\mathbb{R}$ in the sense that each pair of numbers $x, y \in \mathbb{R}$ with $x < y$, one can find $r \in \mathbb{Q}$ such that $x < r < y$. Show that $\mathbb{Q}$ is dense in $\mathbb{R}$ if and only if for every $z \in \mathbb{R}$, there exists a sequence $(r_n)_{n=1}^{\infty}$ such that $r_n \in \mathbb{Q}$ for every $n \in \mathbb{N}$ and $\lim_{n\to\infty} r_n = z$.
3. Let $(a_n)_{n=1}^{\infty}$ be a sequence. Show that $(a_n)_{n=1}^{\infty}$ converges to $A \in \mathbb{R}$ if and only if the subsequences of even and odd terms, $(a_{2n})_{n=1}^{\infty}$ and $(a_{2n-1})_{n=1}^{\infty}$, both converge to $A \in \mathbb{R}$.
4. Let $(a_n)_{n=1}^{\infty}$ be a monotone sequence and suppose that $(a_{n_k})_{k=1}^{\infty}$ is a subsequence of $(a_n)_{n=1}^{\infty}$. Prove that if $(a_{n_k})_{k=1}^{\infty}$ converges to $A \in \mathbb{R}$ then $(a_n)_{n=1}^{\infty}$ converges to A.