HOMEWORK VIII

Due: Thursday, November 16th, 2017.

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1. Prove that if the sequence $(a_n)_{n=1}^{\infty}$ converges to $A \in \mathbb{R}$, then the sequence $\big(|a_n|\big)_{n=1}^{\infty}$ converges to $|A| \in \mathbb{R}$. Provide an example of a sequence which shows that the converse of this statement is not true, in general.

2. Use the ε-N definition for convergent sequences to show the following (*pay close attention to the general form/outline of your proof*):

   (a) If $a \in \mathbb{R}$ then the constant sequence $(a)_{n=1}^{\infty} = (a, a, \dots)$ converges to a.

   (b) $\displaystyle\lim_{n\to\infty} \frac{3n}{n+2} = 3$.

   (c) $\displaystyle\lim_{n\to\infty} \frac{3n+1}{2n+5} = \frac{3}{2}$.

   (d) $\displaystyle\lim_{n\to\infty} \left((-1)^n \frac{n}{2n^2+1}\right) = 0$.

3. Let $(a_n)_{n=1}^{\infty}$ be a sequence and fix a number $A \in \mathbb{R}$. Show that the following three statements are equivalent.

   (a) $\displaystyle\lim_{n\to\infty} a_n = A$.

   (b) $\displaystyle\lim_{n\to\infty} (a_n - A) = 0$.

   (c) $\displaystyle\lim_{n\to\infty} \big|a_n - A\big| = 0$.

4. Let $(a_n)_{n=1}^{\infty}$ be a sequence and fix a number $A \in (0, \infty)$. Show that if $\displaystyle\lim_{n\to\infty} a_n = A > 0$ then there exist $c \in (0, \infty)$ and $N \in \mathbb{N}$ such that $a_n \geq c > 0$ for every $n \in \mathbb{N}$ with $n \geq N$.