HOMEWORK VI

Due: Thursday, November 2, 2017.

Throughout this assignment we assume that X is a nonempty ordered set.

1. Let $\mathbb{F}$ be a field and suppose $x, y \in \mathbb{F}$. Prove the following statements (be sure to justify your argument using field laws, other results we have proved about fields, etc.):
   (a) If $x, y \in \mathbb{F}$ and $x + y = 0$ then $y = -x$.
   (b) If $x \neq 0$ then $-x \neq 0$.
2. Let $\mathbb{F}$ be an ordered field and suppose $x, y, z \in \mathbb{F}$. Prove the following statements (be sure to justify your argument using field laws, other results we have proved about fields, etc.):
   (a) If $x \neq 0$, then $x^2 > 0$.
   (b) If $x > 0$ and $y > 0$, then $x + y > 0$.
3. Let $A \subseteq X$ be a nonempty set which is bounded from above in X and suppose that $b \in X$ is an upper bound for A with the property that $b \in A$. Show that $b = \sup(A)$.
4. Let $A, B \subseteq X$ be two nonempty sets which are bounded from above in X.
   (a) Show that the union $A \cup B$ is bounded from above in X.
   (b) Suppose that $\sup(A), \sup(B) \in X$ exist. Show that $\sup(A \cup B) = \max\big\{\sup(A), \sup(B)\big\}$.
5. Suppose $A \subseteq X$ is a nonempty finite set. Show that A is bounded from above in X. Also, show that $\sup(A)$ exists and $\sup(A) \in A$.

   HINT: Use PMI on the number of elements in A. Also keep in mind that the previous question implies $\sup(S \cup \{x\}) = \max\big\{\sup(S), \{x\}\big\}$ *for any set $S \subseteq X$ and any element $x \in X$.*