# HOMEWORK IV

Due: Tuesday, October 3, 2017.

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1. Let $f : A \to B$ and $g : B \to C$ be two functions. Prove or disprove (with a counterexample) the following statements.
   (a) If $f$ and $g$ are surjective then $g \circ f : A \to C$ is surjective.
   (b) If $g \circ f : A \to C$ is surjective then $f$ is surjective.
   (c) If $g \circ f : A \to C$ is surjective then $g$ is surjective.
2. Let $f : A \to B$ be a given function. Prove that $f$ is injective if and only if for every $y \in B$, either $f^{-1}(\{y\}) = \emptyset$ or $f^{-1}(\{y\})$ contains only one element
3. Suppose that $A$ is a finite set with $n$ elements for some $n \in \mathbb{N}$. Prove that if $x \in A$ then the set $A \setminus \{x\}$ has $n - 1$ elements. Show that this conclusion is not true if $x \notin A$.
4. Suppose that $A$ and $B$ are two disjoint sets and for $n, m \in \mathbb{N}$, fixed, assume that $f : \mathbb{N}_n \to A$ and $g : \mathbb{N}_m \to B$ are two bijective functions. Consider the function $h : \mathbb{N}_{n+m} \to A \cup B$ defined by setting for each $x \in \mathbb{N}_{n+m}$,

$$h(x) := \begin{cases} f(x) & \text{if } x \in \{1, 2, \dots, n\} = \mathbb{N}_n, \\ g(x-n) & \text{if } x \in \{n+1, n+2, \dots, n+m\} = n + \mathbb{N}_m, \end{cases} .$$

   (a) Show that $h$ is a well-defined function. That is, show that the range of $h$, $R(h)$ is a subset of $A \cup B$ and that whenever $h(x) = y$ and $h(x) = z$ with $x \in \mathbb{N}_{n+m}$ and $y, z \in A \cup B$, then $y = z$.
   (b) Show that $h$ is bijective.
   (c) Do you need the hypotheses $A \cap B = \emptyset$? Where, if anywhere, did you use it?