# HOMEWORK X

Due: Monday, December 11th, 2017.

---

1. Determine if each recursively defined sequence $(a_n)_{n=1}^{\infty}$ converges or diverges. In the case of convergence, find its limit and be sure to state if you have used the Monotone Convergence Theorem.
   (a) $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{2}a_n$ for all $n \in \mathbb{N}$.
2. Using the Cauchy sequence definition, show that the sequence $(a_n)_{n=1}^{\infty}$ (as given) is Cauchy.
   (a) $a_n = \dfrac{\sin(n)}{n}$, $n \in \mathbb{N}$.
3. Suppose that $(a_n)_{n=1}^{\infty}$ is a Cauchy sequence and that the set $\{a_n : n \in \mathbb{N}\}$ is finite. Show that $(a_n)_{n=1}^{\infty}$ is eventually a constant sequence.
4. Consider the sequence defined by setting $a_n = \sqrt{n}$ for every $n \in \mathbb{N}$.
   (a) Show that for every $\varepsilon \in (0, \infty)$ there exists $N \in \mathbb{N}$ such that $|a_n - a_{n+1}| < \varepsilon$ for every $n \in \mathbb{N}$ with $n \geq N$.
   (b) Show that $(a_n)_{n=1}^{\infty}$ is not Cauchy.