

# HOMEWORK I

Due: Tuesday, September 12, 2017.

---

Throughout this assignment, suppose that  $A$ ,  $B$ , and  $C$  are sets.

- (a) Show that  $(B \setminus A) \cap (C \setminus A) = (B \cap C) \setminus A$ .  
(b) Show that  $(B \setminus A) \cup (C \setminus A) = (B \cup C) \setminus A$ .
- The **symmetric difference** of  $A$  and  $B$  (written  $A \Delta B$ ) is the set defined as

$$A \Delta B := (A \setminus B) \cup (B \setminus A).$$

Show that  $A \Delta B = (A \cup B) \setminus (A \cap B)$ .

- Given sets  $B$  and  $A_1, A_2, A_3, \dots$ , show that

$$B \cap \left( \bigcup_{n=1}^{\infty} A_n \right) = \bigcup_{n=1}^{\infty} (B \cap A_n) \quad \text{and} \quad B \cup \left( \bigcap_{n=1}^{\infty} A_n \right) = \bigcap_{n=1}^{\infty} (B \cup A_n).$$

- Show that  $A \subseteq B$  if and only if  $A \cup B = B$ .
- Suppose  $n \in \mathbb{Z}$ . If  $n$  is even then  $n^2 - 6n + 5$  is odd.
- Show that if  $A \subseteq B$  then  $A \cap C \subseteq B \cap C$ . What is the converse statement of this implication? Is it true or false? Prove if true and find a counterexample if false.
- For each fixed  $n \in \mathbb{N}$ , consider the set  $A_n := \{(n+1)k : k \in \mathbb{N}\}$ .
  - $A_7 \subsetneq A_3$ .
  - Compute  $\bigcup_{n=1}^{\infty} A_n$  and  $\bigcap_{n=1}^{\infty} A_n$ . Be sure to prove your result.