Analysis on Metric Spaces Conference

Department of Mathematics, University of Pittsburgh

March 10-11, 2017

Friday, March 10th
9:00AM - 9:50AM
Main Speaker

Gareth Speight (University of Cincinnati)

Porosity and Differentiability in Euclidean Spaces and Carnot Groups

Abstract: Rademacher's theorem states that Lipschitz functions between Euclidean spaces are differentiable almost everywhere. Investigating validity of a converse to Rademacher's theorem leads to the construction of small universal differentiability sets, which contain points of differentiability for all Lipschitz functions. Porous sets are sets with relatively large holes on arbitrarily small scales, and have applications to the study of differentiability. For instance, a universal differentiability set cannot be a countable union of porous sets. We discuss measure zero universal differentiability sets in the Heisenberg group and applications of porosity to differentiability in Carnot groups, which can be used to help prove Pansu's differentiability theorem for Lipschitz mappings.

Friday, March 10th
10:00AM - 10:50AM
Main Speaker

Robert Young (New York University)

Embeddings of the Heisenberg group and the Sparsest Cut problem

Abstract: (joint work with Assaf Naor) The Heisenberg group $\mathbb{H}$ is a sub-Riemannian manifold that is hard to embed in $\mathbb{R}^n$. Cheeger and Kleiner introduced a new notion of differentiation that they used to show that it does not embed nicely into L_1. This notion is based on surfaces in $\mathbb{H}$, and in this talk, we will describe new techniques that let us quantify the "roughness" of such surfaces, find sharp bounds on the distortion of embeddings of $\mathbb{H}$, and estimate the accuracy of an approximate algorithm for the Sparsest Cut Problem.

Friday, March 10th
11:15AM - 11:35AM
Contributed Talk

Anton Lukyanenko (University of Michigan)

Quasiconvexity in the Heisenberg group

Abstract: We show that if A is a closed subset of the Heisenberg group whose vertical projections are nowhere dense, then the complement of A is quasiconvex. In particular, closed sets which are null sets for the cc-Hausdorff 3-measure have quasiconvex complements. Conversely, we exhibit a compact totally disconnected set of Hausdorff dimension three whose complement is not quasiconvex.

Friday, March 10th
11:40AM - 12:00PM
Contributed Talk

Scott Zimmerman (University of Pittsburgh)

Sobolev extensions of Lipschitz mappings into metric spaces

Abstract: Wenger and Young proved that the pair $(\mathbb{R}^m, \mathbb{H}^n)$ has the Lipschitz extension property for $m \leq n$ where $\mathbb{H}^n$ is the sub-Riemannian Heisenberg group. That is, for some $C > 0$, any L-Lipschitz map from a subset of $\mathbb{R}^m$ into $\mathbb{H}^n$ can be extended to a CL-Lipschitz mapping on $\mathbb{R}^m$. In this talk, I construct Sobolev extensions of such Lipschitz mappings with no restriction on the dimension m. I will show that any Lipschitz mapping from a compact subset of $\mathbb{R}^m$ into $\mathbb{H}^n$ may be extended to a Sobolev mapping on any bounded domain containing the set. More generally, I will explain this result in the case of mappings into any Lipschitz (n-1)-connected metric space.

Friday, March 10th
1:30PM - 2:20PM
Main Speaker

Sean Li (University of Chicago)

Singular integrals on Heisenberg curves

Abstract: In 1977, Calderon proved that the Cauchy transform is bounded as a singular integral operator on the L_2 space of Lipschitz graphs in the complex plane. This subsequently sparked much work on singular integral operators on subsets of Euclidean space. It is now known that the boundedness of singular integrals of certain odd kernels is intricately linked to a rectifiability structure of the underlying sets. We study this connection between singular integrals and geometry for 1-dimensional subsets of the Heisenberg group where we find a similar connection. However, the kernels studied turn out to be positive and even, in stark contrast with the Euclidean setting. Joint work with V. Chousionis.

Friday, March 10th
2:30PM - 3:20PM
Main Speaker

Vasileios Chousionis (University of Connecticut)

Quantitative rectifiability and intrinsic Lipschitz graphs in the Heisenberg group

Abstract: Rectifiable sets extend the class of surfaces considered in classical differential geometry; while admitting a few edges and sharp corners, they are still smooth enough to support a rich theory of local analysis. However for certain questions of more global nature the notion of rectifiability is too qualitative. In a series of influential papers around the year 1990, G. David and S. Semmes developed an extensive theory of quantitative rectifiability in Euclidean spaces. A motivation for their efforts was the significance of a geometric framework for the study of certain singular integrals and their connections to removability. We will discuss recent efforts towards a theory of quantitative rectifiability in the Heisenberg group. As in the Euclidean case motivation stems from questions involving singular integrals and removability. However new phenomena arise, which do not exist in the Euclidean setting. Based on joint work with K. Fassler and T. Orponen.

Friday, March 10th
3:30PM - 4:20PM
Main Speaker

Mario Bonk (University of California, Los Angeles)

Quasisymmetric uniformization

Abstract: To understand the quasiconformal geometry of a non-smooth or fractal space, one often wants to map it to a less fractal standard space by a quasisymmetry. This quasisymmetric uniformzation problem is related to questions in geometric group theory and complex dynamics, for example. In my talk I will give a survey on some recent developments in this area.

Saturday, March 11th
9:30AM - 10:20AM
Main Speaker

Jeremy Tyson (University of Illinois Urbana-Champaign)

Heisenberg quasiregular ellipticity

Abstract: A Riemannian n-manifold M is quasiregularly elliptic if there exists a nonconstant quasiregular mapping $f : \mathbb{R}^n \to M$. Work of Rickman, Heinonen, Bonk, Pankka, Rajala and many others has greatly clarified the notion of quasiregular ellipticity in the Riemannian context. We initiate the study of Heisenberg quasiregularly elliptic manifolds, sub-Riemannian contact manifolds which receive a nonconstant quasiregular mapping from the Heisenberg group $\mathbb{H}^n$ equipped with its standard sub-Riemannian metric. Pankka and Rajala showed that a 3-manifold link complement admits a Riemannian metric which is quasiregularly elliptic if and only if the link is either empty, the unknot, or the Hopf link. We prove an analogous theorem for 3-manifolds equipped with contact (CR) structures and sub-Riemannian metrics. More generally, we provide a restriction on the growth of the fundamental group for sub-Riemannian contact manifolds which are Heisenberg quasiregularly elliptic. This talk is based on joint work with Katrin Fassler and Anton Lukyanenko.

Saturday, March 11th
10:30AM - 11:20AM
Main Speaker

Luca Capogna (Worcester Polytechnic Institute)

Conformal equivalence of visual metrics in pseudoconvex domains

Abstract: We will present recent joint work with Enrico Le Donne (Jyvaskyla) in which we refine estimates introduced by Balogh and Bonk, to show that the boundary extensions of isometries between smooth strongly pseudoconvex domains in C^n are conformal with respect to the sub-Riemannian metric induced by the Levi form. As a corollary we obtain an alternative proof of a result of Fefferman on smooth extensions of biholomorphic mappings between pseudoconvex domains. The proofs are inspired by Mostow's proof of his rigidity theorem and are based both on the asymptotic hyperbolic character of the Kobayashi or Bergman metrics and on the Bonk-Schramm hyperbolic fillings.

Saturday, March 11th
11:45AM - 12:05PM
Contributed Talk

Vyron Vellis (University of Connecticut)

Euclidean sets contained in good surfaces

Abstract: Let A be a set in the n-dimensional Euclidean space E^n. Given a positive integer m less than n, when is it possible to construct a nice map $f : E^m \to E^n$ so that A is contained in its image? In this talk we present sufficient conditions in terms of the geometry of A and its metric dimension which ensure that A is contained in a quasisymmetric m-plane, a bi-Lipschitz m-plane, a Hölder m-plane or a bi-Hölder line. The conditions on dimension are sharp.

Saturday, March 11th
12:10PM - 12:30PM
Contributed Talk

Xiaodan Zhou (Worcester Polytechnic Institute)

Sobolev embedding on a sphere containing an arbitrary Cantor set in the image

Abstract: In this talk, we will construct a large class of pathological n-dimensional topological spheres $f(\mathbb{S}^n)$ whose image contain an arbitrary Cantor set $C \subset \mathbb{R}^{n+1}$ and the topological embedding $f : \mathbb{S}^n \to \mathbb{R}^{n+1}$ is in the Sobolev class $W^{1,n}$. Our construction resembles that of the Alexander horned sphere. Moreover, by choosing the Cantor set to be Antoine's neckless, we show that there are uncountably many "essentially different" Alexander horned sphere in $\mathbb{R}^3$.

Saturday, March 11th
2:00PM - 2:50PM
Main Speaker

Bruce Kleiner (New York University)

Rigidity of quasiconformal and Sobolev mappings between Carnot groups

Abstract: I will discuss some rigidity theorems for quasiconformal and Sobolev mappings between Carnot groups, in particular, for mappings between products. A key role is played by the pullback of differential forms using the Pansu derivative, and its interaction with distributional exterior derivatives. This is joint work with Stefan Muller and Xiangdong Xie.

Saturday, March 11th
3:00PM - 3:50PM
Main Speaker

Valentino Magnani (University of Pisa)

Area of subsets in nilpotent groups

Abstract: We will present an overview of the problem of finding the natural area of subsets, when the underlying metric structure is non-Riemannian. Recent developments and open questions will be also addressed.

Saturday, March 11th
4:15PM - 5:05PM
Main Speaker

Matthew Badger (University of Connecticut)

Structure theorems for Radon measures

Abstract: One goal of geometric measure theory is to understand a measure on a space through its interaction with canonical subsets in the space. A basic illustration of this is the dichotomy between atomic measures and atomless measures: the former "live on" countably many points, while the later give measure zero to any singleton. In joint work with Raanan Schul, we have solved an analogous problem of identifying the 1-rectifiable part of a Radon measure in Euclidean space, which "lives on" countably many rectifiable curves, and the purely 1-unrectifiable part of the measure, which gives measure zero to any rectifiable curve. A new tool that we introduce is an "anisotropic" square function, which allows us to analyze non-doubling Radon measures. I will also talk about forthcoming joint work with Vyron Vellis, in which we examine the structure of Radon measures with respect to Hölder continuous curves.

Saturday, March 11th
5:15PM - 5:35PM
Contributed Talk

Diego Ricciotti (University of Pittsburgh)

Regularity for p-harmonic functions in the Heisenberg group

Abstract: We provide a proof of the $C^{1,\alpha}$-regularity of weak solutions to the p-Laplace equation in the Heisenberg group for $p > 4$.

Saturday, March 11th
5:40PM - 6:00PM
Contributed Talk

Jake Mirra (University of Pittsburgh)

Developments on the Hölder equivalence question for the Heisenberg group

Abstract: In my talk I will discuss a recent development of the analysis of Hölder continuous mappings into the Heisenberg group. This line of research was initiated by Roger Züst with the motivation of answering the celebrated Hölder equivalence problem of Gromov. I will show that this method can be used to prove Gromov's theorem about non existence of Hölder embeddings of manifolds into $\mathbb{H}^n$ if the Hölder continuity exponent is too large. Also I will show numerical evidence for a counterexample to a conjecture of Gromov. The talk will be based on my joint work with P. Hajłasz and A. Schikorra.